Section 15.7: Cylindrical and Spherical Coordinates

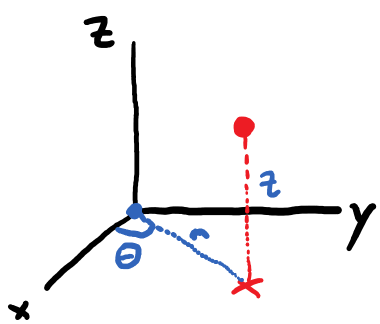
Tuesday, April 7, 2015 3:12 PM

Goals:

1. To convert between rectangular, cylindrical, and spherical coordinates
2. To represent surfaces in space using cylindrical and spherical coordinates

Cylindrical Coordinates

(r, θ, z)



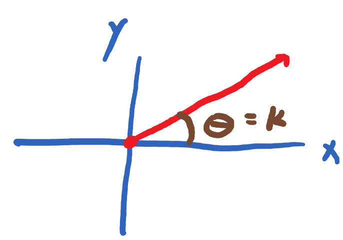
$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ z &= z \\ r^2 &= x^2+y^2 \\ \tan\theta &= \frac{y}{x} \end{aligned}}$$

→ $\boxed{r = a}$, a constant > 0

cylinder

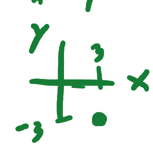
→ $\boxed{\theta = k}$, k constant

half-plance containing z-axis



ex convert to cylindrical coordinates: $(3, -3, -7)$

x y



(r, θ, z)

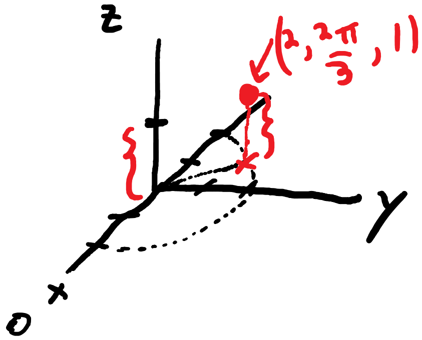
$r = \sqrt{9+(-3)^2} = \sqrt{18} = 3\sqrt{2}$

$\tan\theta = \frac{-3}{3} = -1$

$$\boxed{\left(3\sqrt{2},\ -45^\circ,\ -7\right)}$$

$\theta = -45^\circ$

(ex) Plot $(\overset{r}{2}, \overset{\theta}{\tfrac{2\pi}{3}}, 1)$ in cyl. coord and then convert to rectangular.



$x = r\cos\theta = 2\overbrace{\cos\frac{2\pi}{3}}^{-\frac{1}{2}} = -1$

$y = r\sin\theta = 2\underbrace{\sin\frac{2\pi}{3}}_{\frac{\sqrt{3}}{2}} = \sqrt{3}$

$$\boxed{\left(-1, \sqrt{3}, 1\right)}$$

(ex) Convert to rectangular coordinates and sketch

$r \cdot\ r = 2r\cos\theta$

$r^2 = 2r\cos\theta$

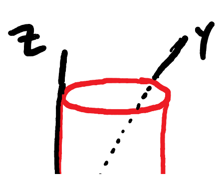
$x^2 + y^2 = 2x$

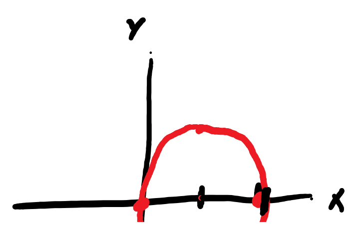
$1x^2 - 2x + 1 + y^2 = 0 + 1$

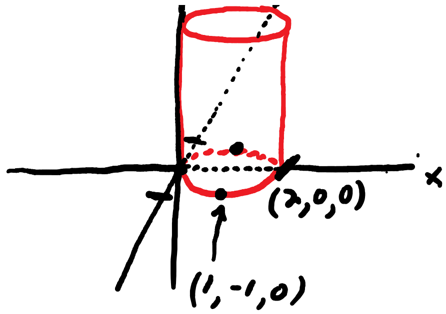
$(x-1)^2 + y^2 = 1$

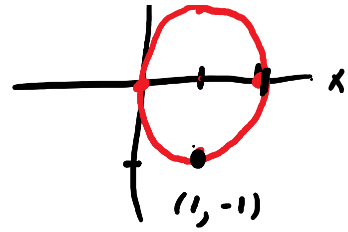
$(x-1)^2 + y^2 = 1$

cylinder w/ axis through $(1,0)$ parallel to z-axis

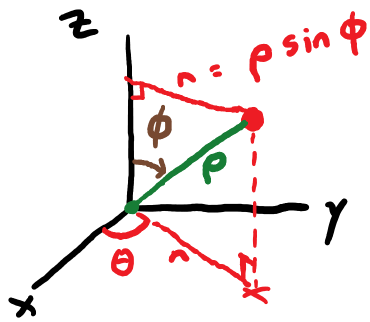








Spherical Coordinates



$0 \le \theta \le 2\pi$

$$(\rho, \theta, \phi)$$

$0 \le \phi \le \pi$

$$x = \overbrace{\rho \sin\phi}^{r} \cos\theta, \quad y = \rho \sin\phi \sin\theta, \quad \boxed{z = \rho\cos\phi}$$

$$\rho^2 = x^2 + y^2 + z^2, \quad \tan\theta = \frac{y}{x}, \quad r = \rho\sin\phi$$

ex convert and I.D.

$x^2 + y^2 - 3z^2 = 0$ cone

$$\underbrace{x^2 + y^2 + z^2}_{\rho^2} - 4 \underset{\uparrow\; z = \rho\cos\phi}{z^2} = 0$$

$\rho^2 - 4$

$$\rho^2 - 4\rho^2\cos^2\phi = 0$$

$$\rho^2(1 - 4\cos^2\phi) = 0$$

$$\rho^2 = 0 \quad \text{or} \quad 1 - 4\cos^2\phi = 0$$

$\rho^2 = 0$ or $1 - 4\cos^2\phi = 0$

$\rho = 0$

$$\cos\phi = \pm\sqrt{\frac{1}{4}}$$

$$\cos\phi = \pm\frac{1}{2}$$

$\cos\phi = -\frac{1}{2}$ or $\cos\phi = \frac{1}{2}$

$\underbrace{\phi = \frac{2\pi}{3}}_{\text{bottom half of cone}}$ $\underbrace{\phi = \frac{\pi}{3}}_{\text{Top half of cone}}$

(ρ, θ, ϕ)

$\rho = a \rightarrow$ sphere

$\theta = b \rightarrow$ half-plane

$\phi = c \rightarrow$ half-cone

ex convert of cylindrical coordinates:

$$\overset{\rho}{(2}, \overset{\theta}{75°}, \overset{\phi}{45°}) = (\sqrt{2}, 75°, \sqrt{2})$$

$$r = \rho\sin\phi = 2\sin 45° = \sqrt{2}$$

$$z = \rho\cos\phi = 2\cos 45° = \sqrt{2}$$