Integration by Parts

Goal: To integrate a wider range of functions using Integration by Parts

$$\boxed{\int u\,dv = uv - \int v\,du}$$ ← integration by parts

Proof:

$$\int \frac{d}{dx}(uv)\,dx = \int\left(\frac{du}{dx}v + \frac{dv}{dx}u\right)dx$$

$$uv = \int\left(v\frac{du}{dx} + u\frac{dv}{dx}\right)dx$$

$$uv = \int v\frac{du}{\cancel{dx}}\cancel{dx} + \int u\frac{dv}{\cancel{dx}}\cancel{dx}$$

$$uv = \int v\,du + \int u\,dv$$

$$uv - \int v\,du = \int u\,dv$$

done.

ex Integrate

a) $\int \underbrace{x}_{u} \underbrace{e^x dx}_{dv}$

$\int u\,dv = \boxed{uv - \int v\,du}$

$u = x \qquad \int dv = \int e^x dx$

$du = dx \qquad v = e^x$

$xe^x - \boxed{\int e^x dx} + C$

$\boxed{xe^x - e^x + C}$

b) $\int x \ln(x^2)\,dx$

$\int u\,dv = \boxed{uv - \int v\,du}$

$u = \ln(x^2) \qquad \int dv = \int x\,dx$

$du = \frac{1}{x^{\cancel{2}}} \cdot 2\cancel{x}\,dx \qquad \boxed{v = \frac{1}{2}x^2}$

$\boxed{du = \frac{2}{x}\,dx}$

$\frac{1}{2}x^2 \ln x^2 - \int \frac{1}{\cancel{2}} x^2 \cdot \frac{\cancel{2}}{x}\,dx + C$

$$\frac{1}{2}x^2\ln x^2 - \int x\,dx + C$$

$$\frac{1}{2}x^2\ln x^2 - \frac{1}{2}x^2 + C$$

c) $\int_0^{\frac{\pi}{2}} \boxed{x^2 \sin x}\,dx$

$\int \underbrace{u}_{\uparrow}\underbrace{dv}_{\uparrow} = uv - \int v\,du$

Time-out

$$\int \underbrace{x^2}_{u} \underbrace{\sin x\,dx}_{dv}$$

$u = x^2, \quad \int dv = \int \sin x\,dx$

$du = 2x\,dx, \quad v = -\cos x$

$$-x^2\cos x - \int(-\cos x)\,2x\,dx$$

$$-x^2\cos x + 2\boxed{\int x'\cos x\,dx}$$

do it again!

$u = x, \quad dv = \cos x\,dx$

(do it again)

$u = x \qquad dv = \cos x \, dx$

$du = dx \qquad v = \sin x$

$x \sin x - \int \sin x \, dx$

$[x \sin x + \cos x]$

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2[x \sin x + \cos x] + C$$

$$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

<u>Time-in</u>

$$\int_0^{\frac{\pi}{2}} x^2 \sin x \, dx = \left[-x^2 \cos x + 2x \sin x + 2 \cos x\right]_0^{\frac{\pi}{2}}$$

$$= \left[\left(0 + \cancel{2} \cdot \frac{\pi}{\cancel{2}} \cdot 1 + 0\right) - (0 + 0 + 2 \cdot 1)\right]$$

$$= \pi - 2$$

d) $\int e^x \sin x \, dx$

$$\int u \, dv = uv - \int v \, du$$

$u = \sin x \qquad dv = e^x \, dx$

$u = \sin x \qquad dv = e^x\,dx$

$du = \cos x\,dx \qquad v = e^x$

$$\boxed{e^x \sin x - \boxed{\int e^x \cos x\,dx} + C}$$

$uv - \int v\,du$

$u = \cos x \qquad dv = e^x\,dx$

$du = -\sin x\,dx \qquad v = e^x$

$$\left[e^x \cos x + \int e^x \sin x\,dx\right]$$

$$\int e^x \sin x\,dx = e^x \sin x - \left[e^x \cos x + \int e^x \sin x\,dx\right] + C_1$$

$$\boxed{1\int e^x \sin x\,dx} = e^x \sin x - e^x \cos x - \boxed{1\cancel{\int e^x \sin x\,dx}} + C_1$$

$$+1\int e^x \sin x\,dx \qquad\qquad +\cancel{\int e^x \sin x\,dx}$$

$$\frac{\cancel{2}\int e^x \sin x\,dx}{\cancel{2}} = \frac{1}{2}e^x \sin x - \frac{1}{2}e^x \cos x + \frac{1}{2}C_1$$

$$= \frac{1}{2}e^x \sin x - \frac{1}{2}e^x \cos x + C$$