Section 6.5: Mean Value Theorem for Integrals and Average Value of a Function

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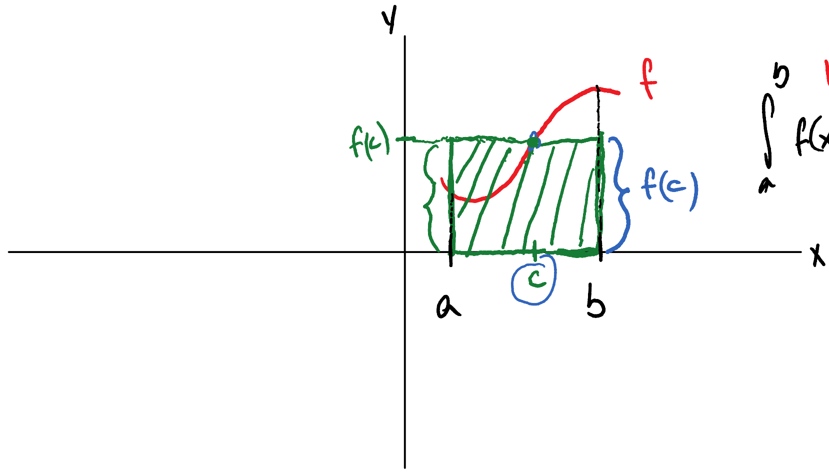


$\int_a^b f(x)\,dx =$ Area from a to b between f and x-axis   ← for $f \geq 0$

Mean Value Theorem for Integrals (MVTI)

For a continuous function, f, there exists a $c \in [a,b]$ such that

$$\boxed{\int_a^b f(x)\,dx = f(c)(b-a)}$$

Average value of f over $[a,b]$

$$\boxed{f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

ex) Let $f(x) = 4 - x^2$

a) f_{ave}

b) Find c such that $f_{ave} = f(c)$

c) sketch f and rectangle whose area is the same as $\int_a^b f(x)\,dx$ where $a=0$, $b=2$.

a) $f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$

$$\int_0^2 (4-x^2)\,dx$$

$$= \left[4x - \frac{x^3}{3}\right]_0^2$$

$$= \left(8 - \frac{8}{3}\right) - 0$$

$b - a$

$2 - 0 = \cancel{4}\ 2$

$f_{ave} = \frac{1}{2}\cdot\frac{16}{3} = \frac{8}{3}$

b)

b)

$f(c) = 4 - c^2$

$4 - c^2 = \frac{8}{3}$

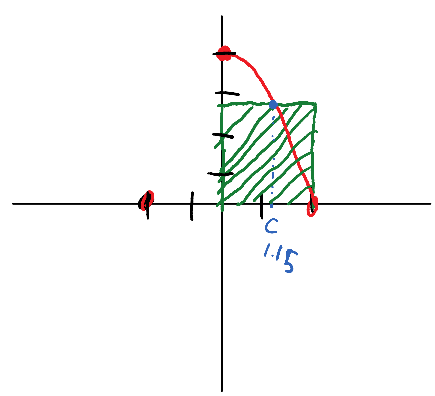
$-c^2 = \frac{8}{3} - 4$

$-c^2 = -\frac{4}{3}$

$\sqrt{c^2} = \pm\sqrt{\frac{4}{3}}$ $\left(a=0,\ b=2 \text{ and } -\frac{2}{\sqrt{3}} \notin [0, 2] \right)$

$c = \cancel{\pm} \frac{2}{\sqrt{3}}$

c) $f(x) = 4 - x^2$



$f\left(\frac{2}{\sqrt{3}}\right) = 4 - \left(\frac{2}{\sqrt{3}}\right)^2$

$= 4 - \frac{4}{3}$

$= \frac{8}{3}$

$\frac{2}{\sqrt{3}} \approx 1.15$