Work

Goal: To quantify the amount of work needed to move an object

Definition: If an object is moved a distance of D in the direction of an applied constant force F, then the work W done by the force is defined as $W = FD$.

Note: There are various types of forces, including electromotive, gravity, and centrifugal. In general, a force can be thought of as a **push** or **pull**.

Example: Find the work done in raising a 120 pound barbell 4 feet.

$$W = F \cdot D$$
$$= 120 \cdot 4$$
$$= 480 \text{ ft} \cdot \text{lb}$$

Note: in the U.S. system of measurement, work is usually expressed in foot-pounds (ft-lb), inch-pounds, or foot-tons. In the centimeter-gram-second (C-G-S) system (or SI metric system) work is expressed in newton-meters (or joules)

In the last example, the force was constant. If the force isn't constant, then the above formula no longer applies. We need calculus to determine the amount of work done if the force is variable.

Suppose that an object is moved along a straight line from x = a to x = b using a variable force. Here's an idea: if we break the interval [a,b] up into n very small subintervals, then the force used to move the object along any one of the n subintervals is close to constant (in other words, the magnitude of the force can't change too awful much over one small subinterval). This means that we can use the formula W = FD to estimate the work done over any "small" subinterval. If we use this formula over all the subintervals and sum the results, then we should get a good estimate of the overall work done. Does this sound familiar?

Let $F(x)$ = Force exerted at position x.

W_i = Work done over the ith subinterval

$$W_i \approx F(c_i)\Delta x$$

Total Work on $[a,b] = W = \lim_{n\to\infty} \sum_{i=1}^{n} F(c_i)\Delta x = \ldots$

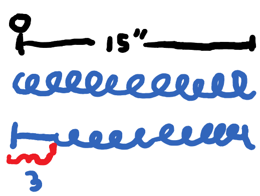
$$W = \int_a^b F(x)\,dx$$

Hooke's Law: The force F used to compress or stretch a spring is directly proportional to the distance *d* that the spring is compressed or stretched from its original length. Symbolically,

F =kd

Where *k* is a constant that depends on the type of spring being compressed or stretched.

Example: A force of 750 pounds compresses a spring 3 inches from its natural length of 15 inches. Find the work done in compressing the spring an additional 3 inches.



$F = kd$

$F = kx$

$750 = k3$

$k = 250$

$F(x) = 250x$

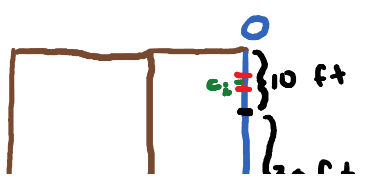
$$W = \int_3^6 250x\,dx$$

$$\left[125x^2\right]_3^6$$

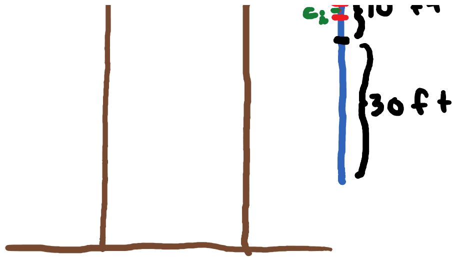
$= 125(36) - 125(9)$

$= 3375$ in·lb

Example: A uniform cable hanging over the edge of a tall building is 40 ft long and weighs 60 lb. How much work is required to pull 10 ft of the cable to the top?



cable weighs $= \dfrac{60 \text{ lb}}{40 \text{ ft}} = \dfrac{3}{2}$ lb/ft



40 ft

Work on 1st 10 ft of cable

$$W_i = F_i \cdot D_i = \underbrace{\left(\tfrac{3}{2}\,\Delta x\right)}_{F_i}\underbrace{c_i}_{D_i}$$

Work on last 30 ft

$W = F \cdot D$

$= \left(\tfrac{3}{2}\cdot 30\right) 10$

$= 450 \text{ ft}\cdot\text{lbs}$

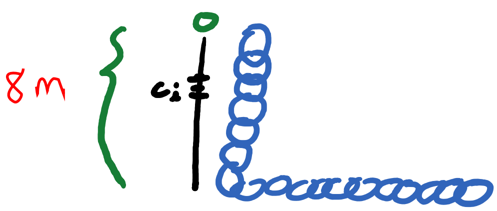
$$W = \frac{3}{2}\int_0^{10} x\,dx$$

$= \frac{3}{2}\cdot\frac{1}{2}\left[x^2\right]_0^{10}$

$= 75 \text{ ft}\cdot\text{lb}$

Total work $= 450 + 75 = 525 \text{ ft}\cdot\text{lb}$

Example: A chain lying on the ground is 20 meters long and its mass is 90 kg. How much work is required to raise one end of the chain to a height of 8 meters?



Force $= (\text{mass})\cdot(\text{Acceleration})$

$\left(\underset{\text{of chain}}{\text{Weight}}\right) = (\text{mass})\cdot(\text{gravity})$

$= (90)\cdot(9.8)$

$= 882 \text{ N}$

Chain weighs $\dfrac{882\text{ N}}{20\text{ m}} = \boxed{44.1\ \text{N/m}}$

$W_i = F_i\, D_i$

$= (44.1\,\Delta x)(8 - c_i)$

$$W = \int_0^8 44.1(8-x)\,dx = 1411.2 \text{ J}$$

Example: A 1600 lb elevator on the end of a 200 ft cable is raised 30 ft. If the cable weighs 2000 lbs, find the work done.

Work ... Elevator | Work on 1st 30 ft

If the cable weighs 2000 lbs, find the work done.

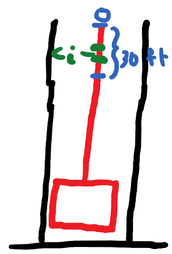
Work on Elevator

$W = 1600 \cdot 30$

$= \boxed{48000 \text{ ft}\cdot\text{lb}}$

Work on 1st 30 ft of cable

Cable weighs $\dfrac{2000 \text{ lbs}}{200 \text{ ft}} = 10 \dfrac{\text{lbs}}{\text{ft}}$



$W_i = F_i \cdot D_i$

$= (10\,\Delta x)\, c_i$

$W = 10\int_0^{30} x\,dx = \boxed{4500 \text{ ft}\cdot\text{lb}}$

Work on last 170 ft of cable

$W = F \cdot D = (10 \cdot 170) \cdot 30$

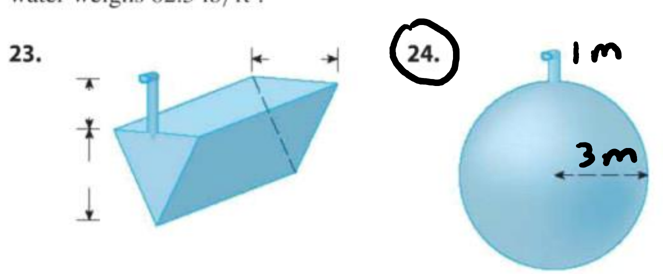
$= 51000 \text{ ft}\cdot\text{lb}$

Total work $= 48000 + 4500 + 51000$ ft·lb

$= 103500$ ft·lb

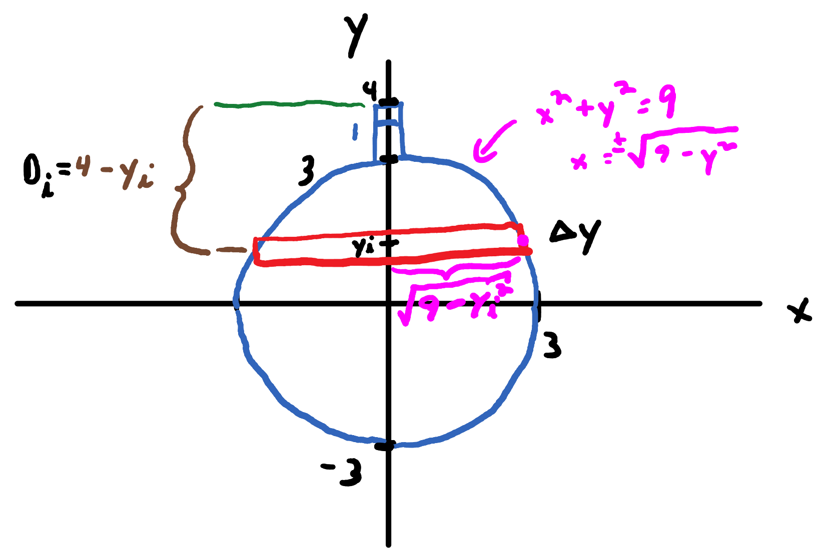
ex

23–26 A tank is full of water. Find the work required to pump the water out of the spout. In Exercises 25 and 26 use the fact that water weighs 62.5 lb/ft^3.



7/8/2015 10:16 PM - Screen Clipping

24 (Stewart 8th Ed ET, Section 6.4)



W_i = Work in pumping out ith slice of water

$W_i = F_i \cdot D_i$

$= \left[\begin{pmatrix} \text{weight} \\ \text{Density} \\ \text{of liquid} \end{pmatrix} \begin{pmatrix} \text{volume} \\ \text{of } i\text{th} \\ \text{slice} \end{pmatrix} D_i \right]$

$$= \left[\underbrace{(\overset{m/v \,\cdot\, g}{1000 \cdot 9.8})}_{\text{wt of 1 m}^3 \text{ of water in N}} \underbrace{\left(\pi \left(\sqrt{9-y_i^2}\right)^2 \Delta y\right)}_{\text{vol.}} \right] \underbrace{(4 - y_i)}_{D_i}$$

$$W = 9800\pi \int_{-3}^{3} (9-y^2)(4-y)\,dy$$

$$= 9800\pi \int_{-3}^{3} (36 - 9y - 4y^2 + y^3)\,dy$$

$$= 9800\pi \left[36y - \frac{9}{2}y^2 - \frac{4}{3}y^3 + \frac{1}{4}y^4 \right]_{-3}^{3}$$

$$= 9800\pi \left[\left(108 - \cancel{\tfrac{81}{2}} - 36 + \cancel{\tfrac{81}{4}}\right) - \left(-108 - \cancel{\tfrac{81}{2}} + 36 + \cancel{\tfrac{81}{4}}\right) \right]$$

$$= 9800\pi \left[216 - 72 \right]$$

$$= 1{,}411{,}200\,\pi \quad \text{J}$$