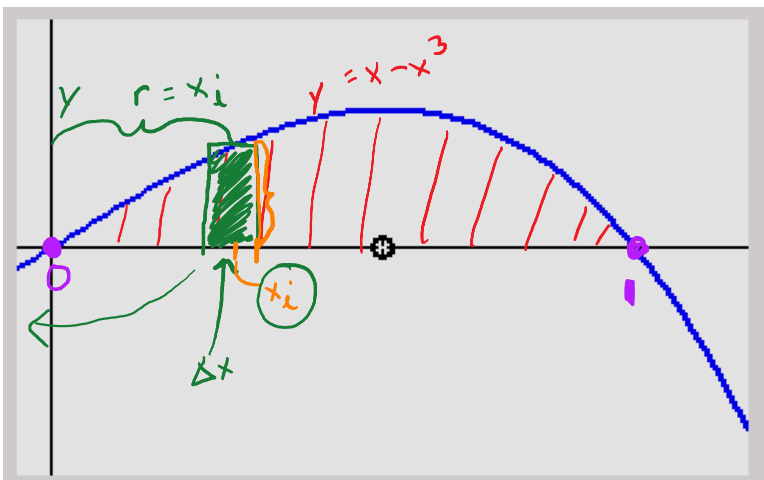
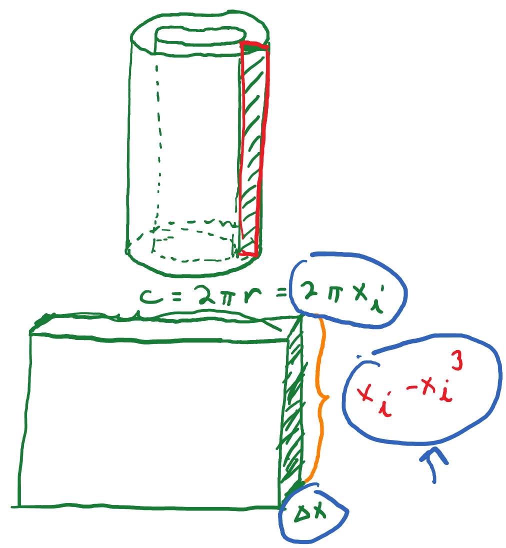
The Shell Method of Finding Volume of Solids of Rotation

Goal: To find the volume of a solid of revolution using the Shell Method.

ex Find the Volume. Rotate about the y-axis:

Region: $y = x - x^3$, $y = 0$, $0 \le x \le 1$





$$V_i = 2\pi x_i (x_i - x_i^3)\Delta x$$

$$V = 2\pi \int_0^1 x(x - x^3)\,dx$$

$$= 2\pi \int_0^1 (x^2 - x^4)\,dx$$

$$= 2\pi \left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1$$

$$2\pi\left[\frac{1}{3} - \frac{1}{5}\right]$$

$$= 2\pi\left[\frac{5}{15} - \frac{3}{15}\right]$$

$$= \frac{4\pi}{15}$$

ex Find the Volume. Rotate about the x-axis:

Region: $x + y = 3$, $x = 4 - (y-1)^2$

x	y
3	0
3	2
0	3
0	-1

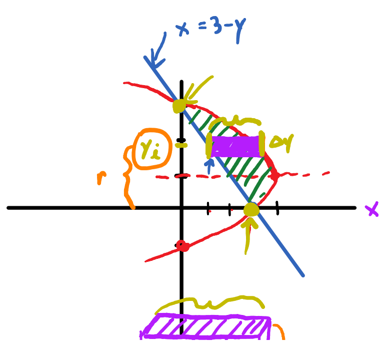
$0 = 4 - (y-1)^2$

$(y-1)^2 = 4$

$y - 1 = \pm 2$

$y = 1 \pm 2$

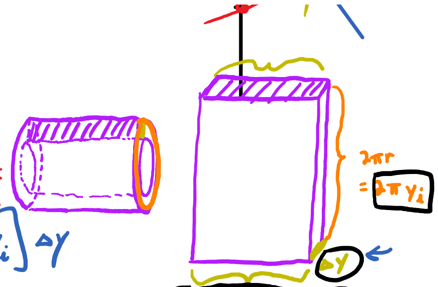
$x + y = 3$



$y = 1 \pm 2$

$x + y = 3$

x	y
3	0
0	3



$2\pi r = \boxed{2\pi y_i}$

$\boxed{4 - (y_i-1)^2 - (3-y_i)}$ $\quad -3+y_i$

$= \boxed{1 - (y_i-1)^2 + y_i}$

$$V_i = 2\pi (y_i)\left[\underbrace{4 - (y_i-1)^2 + y_i}_{}\right]\Delta y$$

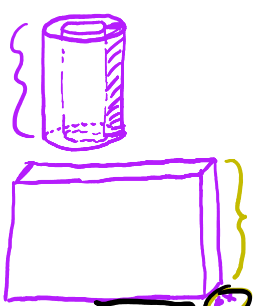
$$2\pi \int_0^3 y\left[3y - y^2\right] dy$$

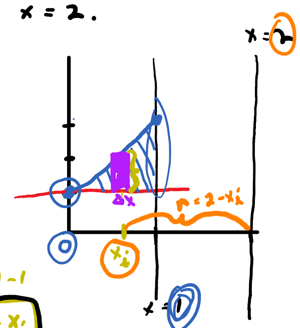
$\vdots$

$= \dfrac{27\pi}{2}$

ex) Find the Volume

Region: $y = x^3 + x + 1$, $\boxed{y=1}$, and $x = 1$. Rotate about the line $x = 2$.





$x_i^3 + x_i + 1 - 1$

$= \boxed{x_i^3 + x_i}$

$C = 2\pi r = \boxed{2\pi(2 - x_i)}$

$$V_i = 2\pi(2 - x_i)(x_i^3 + x_i)\Delta x$$

$$V = 2\pi \int_0^1 (2-x)(x^3+x)\,dx$$

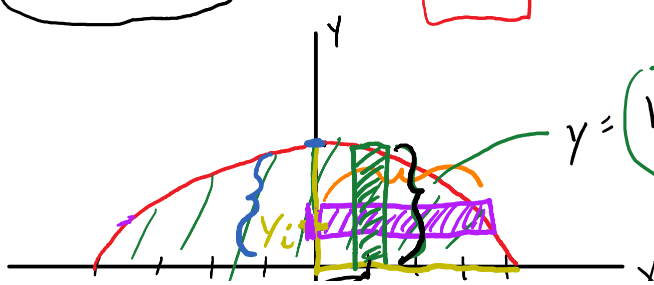
$\vdots$

$= \dfrac{29\pi}{15}$

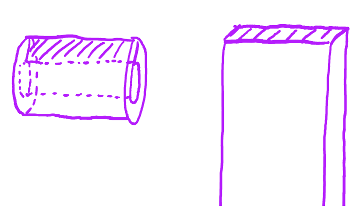
shell

Exercise: A pontoon is to be designed by rotating the graph of $\boxed{y = 1 - \frac{x^2}{16}}$, $-4 \le x \le 4$ about the $\boxed{x\text{-axis}}$. Find the volume of the pontoon.

solve for x.

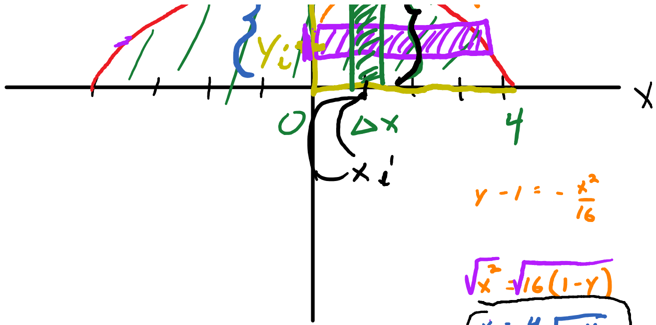


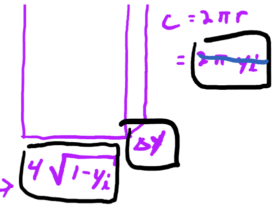
$y = 1 - \dfrac{x^2}{16}$



$c = 2\pi r$

$= \boxed{\cancel{2\pi y_i}}$





$y - 1 = -\frac{x^2}{16}$

$\sqrt{x^2} = \sqrt{16(1-y)}$

$\boxed{x = 4\sqrt{1-y}}$

Disc

$V_i = \pi r^2 h$

$V_i = \pi\left(1 - \frac{x_i^2}{16}\right)^2 \Delta x$

$V = 2\pi \int_0^4 \left(1 - \frac{x^2}{16}\right)^2 dx$

$= \frac{64\pi}{15} \approx 13.4 \text{ ft}^3$

Shell

$V_i =$

$2 \cdot 4 \cdot 2\pi \int_0^1 y\sqrt{1-y}\, dy$

$16\pi \int_0^1 y\sqrt{1-y}\, dy = \frac{64\pi}{15}$