Integration by Substitution

Goal: To evaluate indefinite and definite integrals by substitution

Recall: $\int \frac{d}{dx}\left[F(g(x))\right]dx = \int F'[g(x)]\, g'(x)\,dx = \int f[g(x)]\, g'(x)\,dx$

substitution rule

$$\boxed{F[\overbrace{g(x)}^{u}] + c = \int f[\overbrace{g(x)}^{u}]\, \overbrace{g'(x)\,dx}^{du}}$$

$$\boxed{\int f(u)\,du = F[u] + C}$$

ex) Evaluate using u-substitution

a) $\int \underbrace{(x^2+1)}_{u}{}^2\, \underbrace{2x\,dx}_{du}$

$u = x^2+1$

$\boxed{du = 2x\,dx} \rightarrow dx = \frac{du}{2x}$

$= \int u^2\, \cancel{2x}\, \frac{du}{\cancel{2x}}$

$= \int u^2\,du$

$\frac{(x^2+1)^3}{3} + c$

/

$= \int u^2 \, du$

$= \frac{u^3}{3} + C$

$= \frac{(x^2+1)^3}{3} + C$

b) $\int \sqrt{2x-1} \, dx$

$\underbrace{2x-1}_{u}$, $du = 2dx$

$= \frac{1}{2} \int \sqrt{2x-1} \; 2\,dx$

think $\int \sqrt{u} \, du$

$\int u^{\frac{1}{2}} du$

$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} (2x-1)^{\frac{3}{2}} + C$

$= \frac{1}{3} (2x-1)^{\frac{3}{2}} + C$

c) $\int \sin^2(3x)\underbrace{\cos(3x)\,dx}$

$u = \sin 3x$

$du = 3\cos(3x)\,dx$

$dx = \dfrac{du}{3\cos 3x}$

$\sin^2(3x) = (\underbrace{\sin 3x}_{u})^2$

$du = 3\cos 3x\,dx$

$\int u^2 \cancel{\cos(3x)} \dfrac{du}{3\cancel{\cos(3x)}}$

$\int u^2 \dfrac{du}{3}$

$\dfrac{1}{3} \cdot \dfrac{u^3}{3} + c$

$\dfrac{u^3}{9} + c$

$\dfrac{1}{9}\sin^3(3x) + C$

d) $\frac{1}{2}\int_{x=0}^{x=1} x(\overbrace{x^2+1}^{u})^3 2\,dx$

Time-out

$\int x(\underbrace{x^2+1}_{u})^3\,dx$

$u \quad du = (2)x\,dx$

Alternate Reality:

$\boxed{u = x^2+1}$

$du = 2x\,dx$

$a = 0^2+1 = 1$

$b = 1^2+1 = 2$

$$\int x\,(\underbrace{x^2+1}_{u})\,dx \qquad du = 2x\,dx$$

$$\frac{1}{2}\int \underbrace{(x^2+1)^3}_{u^3}\ \underbrace{2x\,dx}_{du} \qquad \text{(}u^3\ du\text{)}$$

$$= \frac{1}{2}\cdot\frac{1}{4}(x^2+1)^4 + C = \boxed{\frac{1}{8}(x^2+1)^4} + C$$

$\boxed{u = x^2+1}$

$du = 2x\,dx$

$b = 1^2+1 = 2$

$$\frac{1}{2}\int_1^2 u^3\,du$$

Time-in

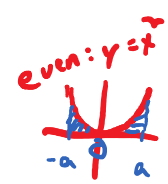
$$\int_0^1 x(x^2+1)^3\,dx = \frac{1}{8}\Big[(x^2+1)^4\Big]_0^1$$

$$= \frac{1}{8}\Big[(1^2+1)^4 - (0^2+1)^4\Big]$$

$$= \frac{1}{8}\big[16-1\big]$$

$$= \frac{15}{8}$$

Note: Integration of Symmetric Functions



① If f is even, then $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

② If f is odd, then $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$