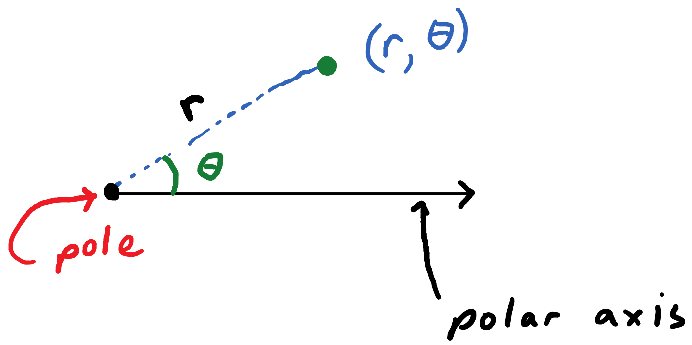
Polar Curves

Goals: To plot curves and take derivatives in polar coordinates.

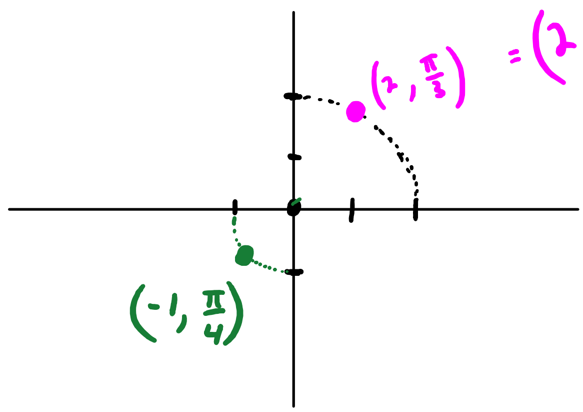
Polar Coordinates



ex) plot the points

a) $(2, \frac{\pi}{3})$ (r, θ)

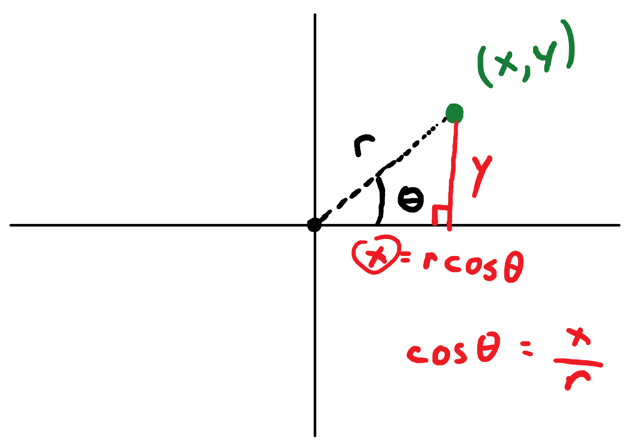
b) $(-1, \frac{\pi}{4})$ (r, θ)



$= (2, \frac{\pi}{3} + 2\pi) = (2, \frac{7\pi}{3})$, where $\frac{\pi}{3} + 2\pi = \frac{6\pi}{3}$ … ($2\pi = \frac{6\pi}{3}$)

coordinate conversions

$$r^2 = x^2 + y^2$$
$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$\tan\theta = \frac{y}{x}$$



ex) convert to rectangular

$(5, \pi)$ (r, θ)

(ex) convert to rectangular

a) $(r,\theta) = \left(5, \frac{\pi}{6}\right)$

$$x = r\cos\theta = 5\cos\frac{\pi}{6} = 5\cdot\frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}$$

$$y = r\sin\theta = 5\sin\frac{\pi}{6} = 5\cdot\frac{1}{2} = \frac{5}{2}$$

$$\left(\frac{5\sqrt{3}}{2}, \frac{5}{2}\right)$$

b) $r = \cos\theta + 2\sin\theta$

$$r\cdot r = r\cdot(\cos\theta + 2\sin\theta)$$

$$r^2 = r\cos\theta + 2r\sin\theta$$

$$x^2 + y^2 = x + 2y$$

$$x^2 - 1x + \frac{1}{4} + y^2 - 2y + 1 = 0 + \frac{1}{4} + 1$$

$$\left(x - \frac{1}{2}\right)^2 + (y-1)^2 = \frac{5}{4}$$

Circle: C $\left(\frac{1}{2}, 1\right)$ radius $= \frac{\sqrt{5}}{2}$

c) convert to polar: $(-1, 1)$

We need (r, θ)

$r^2 = x^2 + y^2$

$r^2 = (-1)^2 + (1)^2$

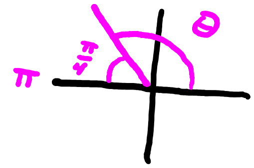
$r^2 = 2$

$r = \sqrt{2}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{1}{-1} = -1$

$\alpha = \frac{\pi}{4}$

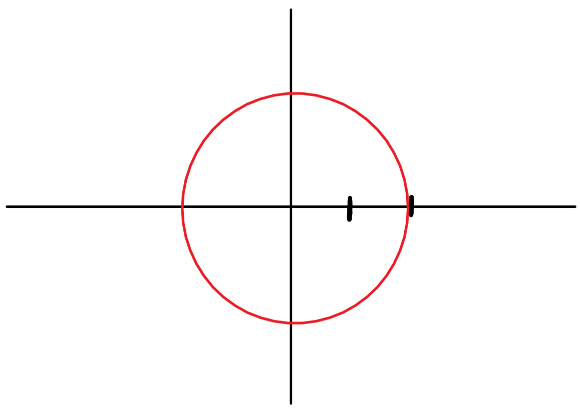


$\theta = \frac{3\pi}{4}$

$\left(\sqrt{2}, \frac{3\pi}{4}\right)$

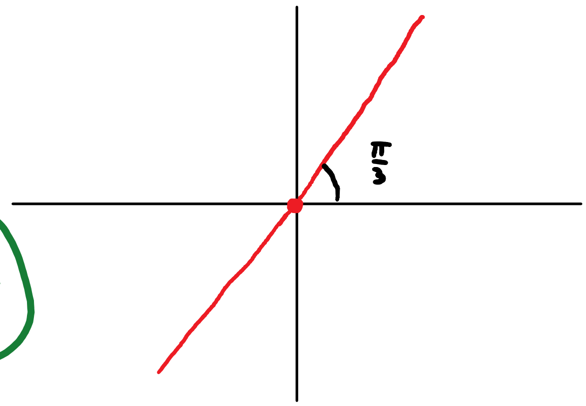
ex Graph

a) $r = 2$



b) $\theta = \frac{\pi}{3}$

$\theta = \frac{\pi}{3}$



$\frac{y}{x} = \tan\theta = \tan\frac{\pi}{3}$

$$\frac{y}{x} = \tan\frac{\pi}{3}$$

$$\frac{y}{x} = \sqrt{3}$$

$$y = \sqrt{3}\,x$$

Special Polar Graphs

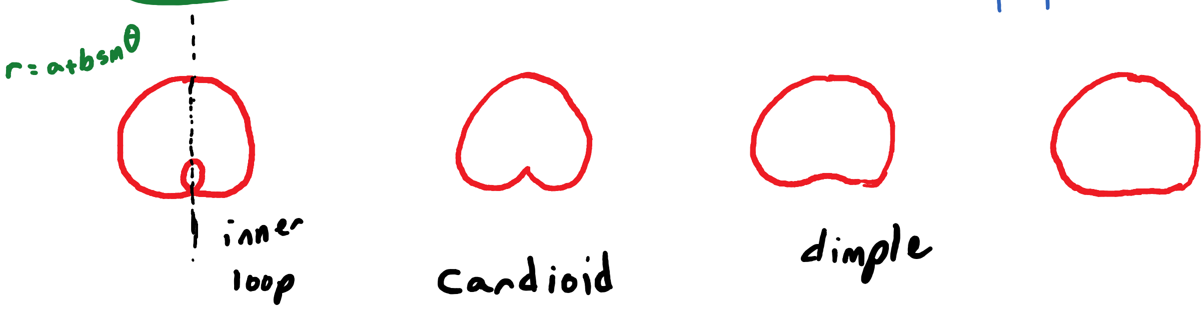
$r = 1 + 2\sin\theta$

$\frac{1}{2}$

Limacon

$r = a \pm b\cos\theta$ or $r = a \pm b\sin\theta$, a and b constants

$0 < \left|\frac{a}{b}\right| < 1$ $\quad\left|\frac{a}{b}\right| = 1$ $\quad 1 < \left|\frac{a}{b}\right| < 2$ $\quad \left|\frac{a}{b}\right| > 2$



Rose Curves

$r = a\sin(n\theta)$ or $r = a\cos(n\theta)$, where

n is a counting number > 1. $(n = 2, 3, 4, \ldots)$

If n is odd → n petals

n is even → $2n$ petals

(ex) Graph

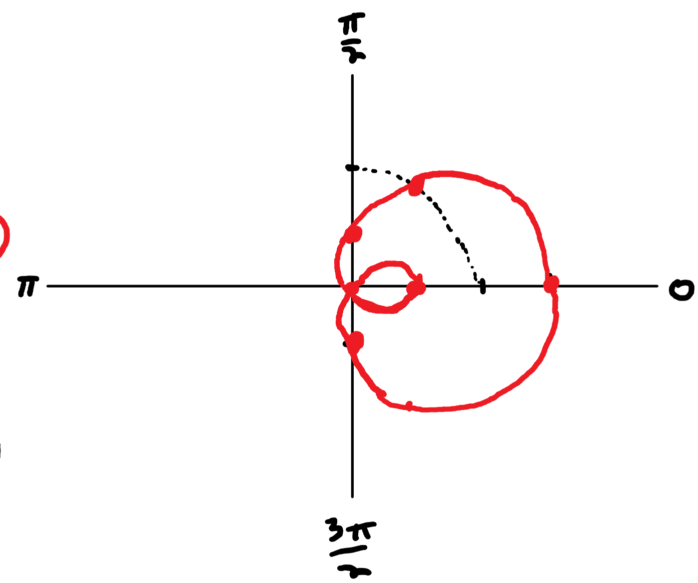
a) $r = 1 + 2\cos\theta$ ← sym to x-axis

limaçon

$a = 1 \quad b = 2$

$\left|\frac{a}{b}\right| = \frac{1}{2}$

r	θ
3	0
2	$\pi/3$
1	$\pi/2$
-1	π
1	$\frac{3\pi}{2}$
3	2π



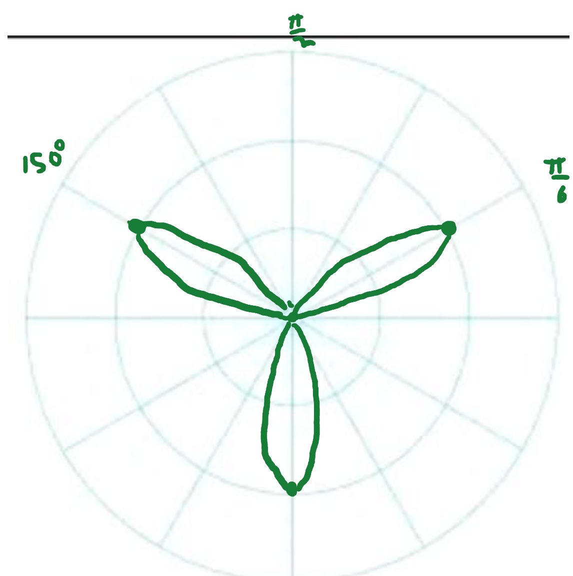
b) $r = 2\sin 3\theta$

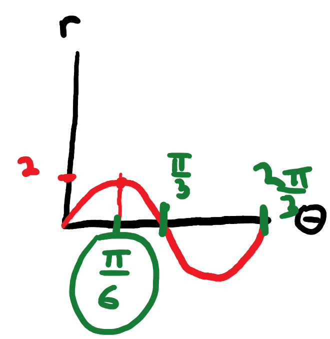
$n = 3$

3 petals

$P = \frac{2\pi}{3}$

$\frac{360^\circ}{3} = 120^\circ$





(ex) Find the slope of the tangent line to (ex) Find the slope of the tangent line to $r = 2\sin\theta$ at $\theta = \frac{\pi}{6}$

$x = f(\theta)$, $y = g(\theta)$ — parametric eqns.

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$x = r\cos\theta$, $y = r\sin\theta$

$x = 2\sin\theta\cos\theta = \sin 2\theta$, $y = 2\sin^2\theta$

$\frac{dy}{d\theta} = 4\sin\theta\cos\theta$

$\frac{dx}{d\theta} = 2(\cos^2\theta - \sin^2\theta)$

$$\frac{dy}{dx} = \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{\sin 2\theta}{\cos 2\theta}$$

$$\frac{dy}{dx} = \tan(2\theta)$$

$$\left.\frac{dy}{dx}\right|_{\theta = \frac{\pi}{6}} = \tan\left(2\left(\frac{\pi}{6}\right)\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

Another way

$x = \sin 2\theta$, $\frac{dx}{d\theta} = 2\cos 2\theta$

$y = 2\sin^2\theta$, $\frac{dy}{d\theta} = 4\sin\theta\cos\theta = 2\sin 2\theta$

$$\frac{dy}{dx} = \frac{2\sin 2\theta}{2\cos 2\theta} = \tan 2\theta$$

easier way →

Notes: ① To find a H-tangent to a polar curve set $dy/d\theta = 0$

② To find a V-tangent set $\frac{dx}{d\theta} = 0$

$$\frac{dx}{d\theta} = 0$$

(3) Be careful if both $\frac{dy}{d\theta} = 0$ and $\frac{dx}{d\theta} = 0$