Section 10.1: Parametric Curves

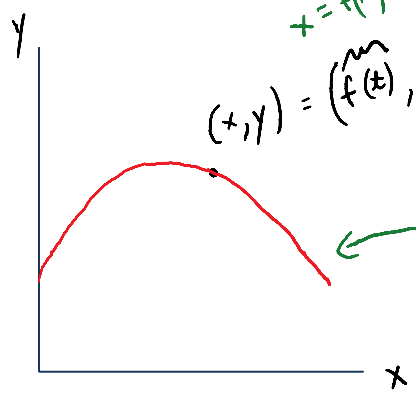
Monday, March 24, 2014
12:29 PM

Goals:

1. To graph parametric curves.
2. To convert between parametric equations and rectangular coordinates.

$x = f(t) \quad y = g(t)$

$(x, y) = (f(t), g(t))$



path of a projectile can be represented parametrically where $(f(t), g(t))$ gives the position of the projectile at time t.

When we graph $y = f(x)$, we plot ordered pairs (x,y). Sometimes it's convenient to think of x and y in terms of a third variable t (called the parameter) so that

$x = f(t)$ and $y = g(t)$. (any letter can be used in place of t).

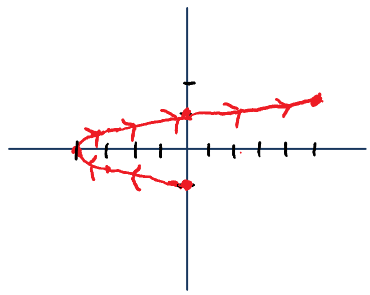
ex Sketch the curve given by

$\underbrace{x = \overbrace{t^2 - 4}^{f(t)}}$ and $\underbrace{y = \overbrace{\frac{t}{2}}^{g(t)}}$, $-2 \le t \le 3$

parametric equations

t	x	y
-2	0	-1
-1	-3	$-1/2$
0	-4	0
1	-3	$\frac{1}{2}$
2	0	1
3	5	$\frac{3}{2}$

$(f(-2),\ g(-2))$



Notes:

(1) For parametric equations $x = f(t)$, $y = g(t)$ with $a \le t \le b$,

$\underbrace{x = f(a) \text{ and } y = g(a)}_{(f(a),\ g(a))}$ called initial point

$\underbrace{x = f(b),\ y = g(b)}_{(f(b),\ g(b))}$

$(f(b),\ g(b))$ called terminal point.

(2) The curve is given by $(f(t), g(t))$ where $a \le t \le b$

$(f(t), g(t))$: set of ordered pairs that gives the curve.

convert to rectangular

ex) sketch by eliminating the parameter

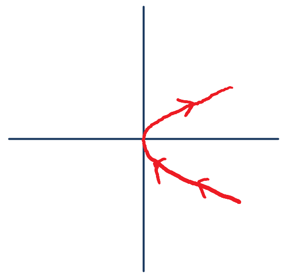
☆ $\underline{\text{trick}}$: solve for t in one of the parametric equations and plug into the other.

a) $x = t^2$, $\underbrace{y = t}_{t=y}$

$$x = y^2$$

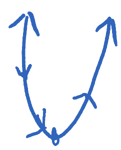
$\underbrace{y = \sqrt{x}}$ top half of parabola

$\underbrace{y = -\sqrt{x}}$ bottom



b) $\underbrace{x = t}_{t=x}$ $y = t^2$

$$y = x^2$$



Note: If $y = f(x)$ let $x = t$, then the parametric eqns

are $x = t$, $y = f(t)$

c) $x = \dfrac{1}{\sqrt{t+1}} > 0$ and $y = \dfrac{t}{t+1}$

☆ solve for t in one of the equations

$(\quad)$

$$x = \frac{1}{\sqrt{t+1}} \quad \Big\} \text{ solve for } t.$$

t	x	y
0	1	0
1	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$

$$x^2 = \frac{1}{t+1}$$

$$t+1 = \frac{1}{x^2}$$

$$t = \frac{1}{x^2} - 1$$

$$t = \frac{1-x^2}{x^2}$$

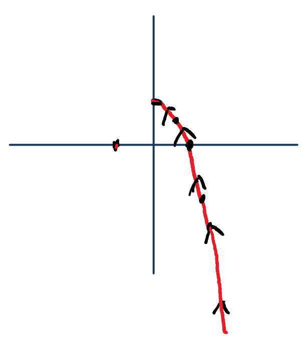
$$y = \frac{\frac{1-x^2}{x^2}}{\left(\frac{1-x^2}{x^2}\right)+1}$$

$$y = \frac{\frac{1-x^2}{x^2}}{\frac{1-x^2+x^2}{x^2}}$$

$$y = \frac{\frac{1-x^2}{x^2}}{\frac{1}{x^2}}$$

$$y = 1-x^2, \quad \boxed{x>0}$$

Domain



t	x	y

t	x	y
0	1	0
1	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$

θ is parameter

(ex) Sketch $\underbrace{x = 3\cos\theta}_{\cos\theta = \frac{x}{3}}$, $\underbrace{y = 3\sin\theta}_{\sin\theta = \frac{y}{3}}$, $0 \le \theta \le 2\pi$

trig identity $\boxed{\cos^2\theta + \sin^2\theta = 1}$

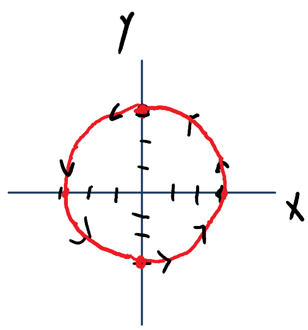
sub in here

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$

$$\frac{x^2}{9} + \frac{y^2}{9} = 1$$

$$x^2 + y^2 = 9$$

circle, $r = 3$, $C(0,0)$



θ	x	y
0	3	0
$\frac{\pi}{2}$	0	3

(ex) Rectangularlize

$x = 2 + 3\cos\theta$, $y = 3 + \sin\theta$ $\qquad 0 \le \theta \le 2\pi$

$\cos\theta = \frac{x-2}{3}$, $\sin\theta = y - 3$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\left(\frac{x-2}{3}\right)^2 + (y-3)^2 = 1$$

$$\frac{(x-2)^2}{9} + \frac{(y-3)^2}{1^2} = 1$$

Ellipse with center (2,3)