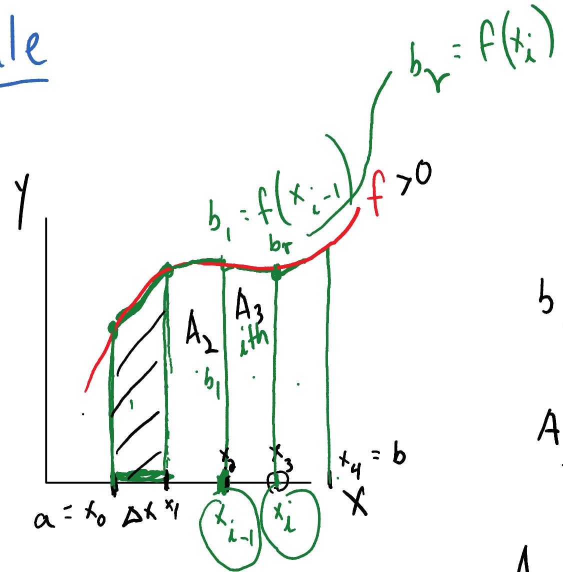
Numerical Integration

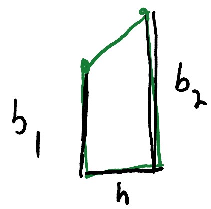
Wednesday, February 19, 2014

12:41 PM

Goal: To use the Trapezoid and Simpson Rules to approximate definite integrals.

Trapezoid Rule





$$A_{trap} = \frac{1}{2}(b_1 + b_2)h$$

$$A_i = \frac{1}{2}\left(f(x_{i-1}) + f(x_i)\right)\Delta x$$

$$A_1 = \frac{1}{2}\left(f(x_0) + f(x_1)\right)\Delta x$$

$$A_2 = \frac{1}{2}\left(f(x_1) + f(x_2)\right)\Delta x$$

$$A_3 = \frac{1}{2}\left(f(x_2) + f(x_3)\right)\Delta x$$

$$\vdots$$

$$A_n = \frac{1}{2}\left(f(x_{n-1}) + f(x_n)\right)\Delta x$$

$$\text{Area} = \frac{1}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + \cdots + 2f(x_{n-1}) + f(x_n)\right]\Delta x$$

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + f(x_n)\right]$$

Trapezoid Rule

(ex) Use the trapezoid rule with $n=4$ to estimate

$$\int_2^3 \frac{1}{x^2-1}\,dx$$

f

$\Delta x = \frac{b-a}{n} = \frac{3-2}{4} = \frac{1}{4} = 0.25$ | endpts $2, 2.25, 2.5, 2.75, 3$

$\frac{b-a}{2n} = \frac{1}{2\cdot 4} = \frac{1}{8} = 0.125$

$$\int_2^3 \frac{1}{x^2-1}\,dx \approx 0.125\left[f(2) + 2f(2.25) + 2f(2.5) + 2f(2.75) + f(3)\right]$$

$$= 0.125\left[\frac{1}{3} + \frac{2}{2.25^2-1} + \frac{2}{2.5^2-1} + \frac{2}{2.75^2-1} + \frac{1}{8}\right]$$

$$= 0.2045444139$$

Simpson's Rule

$$\int_a^b f(x)\,dx \approx \frac{b-a}{3n}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)\right]$$

where n is even.

(ex) est. $\int_0^\pi \sin x^2\,dx$ with $n=4$.

f

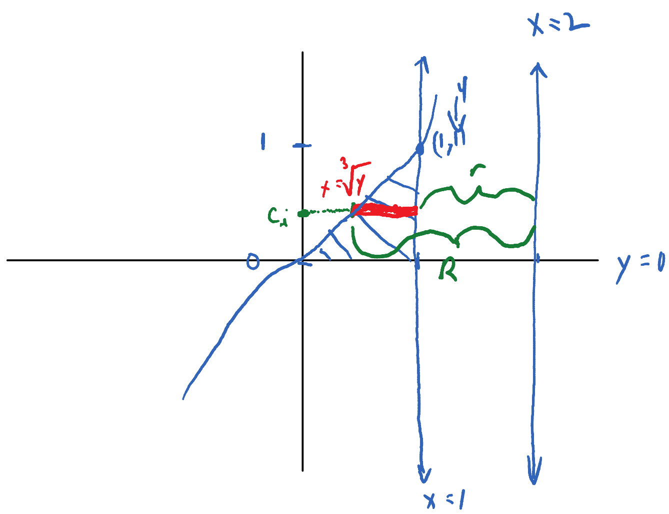
$$\Delta x = \frac{b-a}{n} = \frac{\pi - 0}{4} = \frac{\pi}{4} \quad \Big| \quad \text{endpts: } 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$$

$$\int_0^{\pi} \sin x^2\,dx \approx \frac{\pi}{12}\left[\sin(0^2) + 4\sin\left(\frac{\pi^2}{16}\right) + 2\sin\left(\frac{\pi^2}{4}\right) + 4\sin\left(\frac{9\pi^2}{16}\right) + \sin(\pi^2)\right]$$

$$= 0.12044$$

6.2

(15) $\underbrace{y = x^3}_{x = \sqrt[3]{y}}$, $y = 0$, $x = 1$; about $x = 2$



$R = 2 - \sqrt[3]{c_i}$

$r = 2 - 1 = 1$

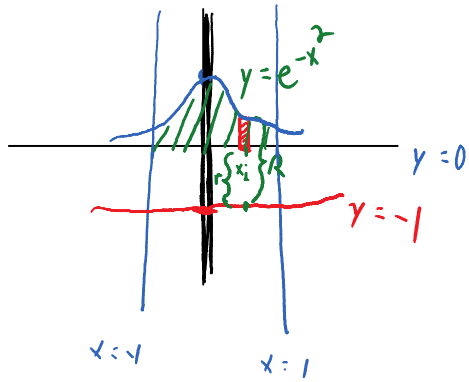
$$V_i = \pi R^2 h - \pi r^2 h$$

$$= \pi\left(2 - \sqrt[3]{c_i}\right)^2 \Delta y - \pi\left(1\right)^2 \Delta y$$

$$V = \pi \int_0^1 \left[\left(2 - \sqrt[3]{y}\right)^2 - 1\right] dy$$

(31) $y = e^{-x^2}$, $y = 0$, $x = -1$, $x = 1$

b) $y = -1$



$R = e^{-x_i^2} - (-1)$

$= e^{-x_i^2} + 1$

$r = 0 - (-1) = 1$

$V_i = \pi R^2 h - \pi r^2 h$

$= \pi \left(e^{-x_i^2} + 1\right)^2 \Delta x - \pi 1^2 \Delta x$

$V = 2\pi \int_0^1 \left[\left(e^{-x^2} + 1\right)^2 - 1\right] dx$