# Homework Questions

Wednesday, January 29, 2014
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4.4

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air resistance

$$v = \frac{mg}{c}\left(1 - e^{-\frac{ct}{m}}\right), \quad r = \cancel{c}^{\,0^+} v \to 0^+$$

as $c \to 0^+$

a) $$\lim_{t\to\infty} \frac{mg}{c}\left(1 - \cancel{e^{-\frac{ct}{m}}}^{\,0 \text{ as } t\to\infty}\right) = \frac{mg}{c}$$

terminal velocity of object

b) $$\lim_{c\to 0^+} \cancel{\frac{mg}{c}}^{\,\infty}\left(1 - \cancel{e^{-\frac{ct}{m}}}^{\,1}\right) \qquad \infty \cdot 0$$

$$\cancel{\lim_{c\to 0^+}\left(\frac{mg}{c} - \frac{mg}{c} e^{-\frac{ct}{m}}\right)}$$

$$= \lim_{c\to 0^+} \frac{mg \cdot \left(1 - \cancel{e^{-\frac{ct}{m}}}^{\,1}\right)}{c} \qquad \frac{0}{0} \checkmark$$

H

$$= \lim_{c\to 0^+} \frac{\cancel{m}g\frac{t}{\cancel{m}}\, \cancel{e^{-\frac{ct}{m}}}^{\,1}}{1}$$

$$= gt.$$

So as $c \to 0^+$, air resistance goes to $0^+$ and and the speed of the object is independent of mass and equals $gt$. Thus, in a vacuum, $v = gt$ and is independent of mass.

(29) $\int \ln\left(x+\sqrt{x^2-1}\right) dx$

$\frac{x}{1} = \sec\theta$ $\quad \frac{1}{x} = \cos\theta$

$dx = \sec\theta \tan\theta \, d\theta$

$\int \ln\left(\sec\theta + \sqrt{\underbrace{\sec^2\theta - 1}_{\tan^2\theta}}\right) \sec\theta\tan\theta \, d\theta$

$\int \underbrace{\ln\left(\sec\theta + \tan\theta\right)}_{u} \underbrace{\sec\theta \tan\theta \, d\theta}_{dv}$ $\qquad \int u \, dv = uv - \int v \, du$

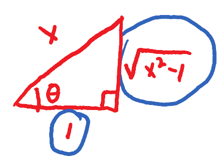
$u = \ln\left(\sec\theta + \tan\theta\right)$ $\qquad \int dv = \int \sec\theta \tan\theta \, d\theta$

$du = \sec\theta$ $\qquad v = \sec\theta$

$\sec\theta \ln(\sec\theta + \tan\theta) - \int \sec^2\theta \, d\theta$

$\sec\theta \ln(\sec\theta + \tan\theta) - \tan\theta + c$

$x \ln\left(x + \sqrt{x^2-1}\right) - \sqrt{x^2-1} + c$




(37) $\int_0^{\frac{\pi}{4}} \tan^3\theta \sec^2\theta \, d\theta$

time-out

$\int \tan^3\theta \underbrace{\sec^2\theta \, d\theta}_{du}$

$u = \tan\theta$

$du = \sec^2\theta \, d\theta$

$du = \sec^2\theta \, d\theta$

$$\int u^3 \, du$$

$$\frac{u^4}{4} + c$$

$$\frac{\tan^4\theta}{4} + c$$

time-in

$$= \frac{1}{4}\left[\tan^4\theta\right]_0^{\pi/4}$$

$$= \frac{1}{4}\left[1 - 0\right]$$

$$= \frac{1}{4}$$

PFD

$$x(x-1)(x+2)^2\left(\frac{x+1}{x(x-1)(x+2)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+2} + \frac{D}{(x+2)^2}\right)$$

$$x+1 = A(x-1)(x+2)^2 + Bx(x+2)^2 + Cx(x-1)(x+2) + Dx(x-1)$$

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$$= \int \frac{4x}{(x+1)(x^2+1)}\,dx$$

$x^3+$

$x^3+x^2+(x+1)\cdot 1$

$x^2(x+1)+1\cdot(x+1)$

$(x^2+1)(x+1)$

PFD

$$\frac{4x}{(x^2+1)(x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$4x = A(x^2+1) + (Bx+C)(x+1)$$

shortcut

$\boxed{x=-1}$

$-4 = 2A$

$A = -2$

Let $x=0$

$0 = -2 + C$

$C = 2$

let $x=1$

$4 = -4 + (B+2)2$

$4\,\cancel{8} = B+2$

$B = 2$

PFD $\dfrac{1}{x+1} + \dfrac{2x+2}{x^2+1}$

## Alternative Method

$$4x = A(x^2+1) + (Bx+C)(x+1)$$

$$4x = Ax^2 + A + Bx^2 + Bx + Cx + C$$

$$" = \underbrace{Ax^2 + Bx^2} + \underbrace{Bx + Cx} + A + C$$

$$4x = (A+B)x^2 + (B+C)x + (A+C)$$

$$\left.\begin{aligned} A + B &= 0 \\ B + C &= 4 \\ A + C &= 0 \end{aligned}\right\}$$

$C = -A$

$$\begin{aligned} A + B &= 0 \\ -A + B &= 4 \\ \hline 2B &= 4 \end{aligned}$$

$B = 2$

$A = -2$

$C = 2$