Wednesday, April 23, 2014
1:24 PM

11.5

(15) $\sum_{n=0}^{\infty} \frac{\sin\left[\left(n+\frac{1}{2}\right)\pi\right]}{1+\sqrt{n}}$ ← should have had these brackets

Note that $\sin\left[\left(n+\frac{1}{2}\right)\pi\right]$ alternates between 1 and -1 for $n = 0, 1, 2, 3, \ldots$

So, this is an alternating series with $b_n = \frac{1}{1+\sqrt{n}} > 0$.

(1) $1 > \frac{1}{2} > \frac{1}{1+\sqrt{3}} > \cdots$

so, b_n decreases

(2) $\frac{1}{1+\sqrt{n}} \to 0$.

So, the series converges by AST.

11.4

(29) $\sum_{n=1}^{\infty} \frac{1}{n!}$

$$n! = \underbrace{n(n-1)(n-2)\cdots 2\cdot 1}_{n \text{ factors } (n-1 \text{ of them } \geq 2)} \boxed{\geq} 2^{n-1}$$

So, $0 \leq \frac{1}{n!} \leq \frac{1}{2^{n-1}}$

Now, $\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1}$ is

a convergent geometric series $\left(-1 < r = \frac{1}{2} < 1\right)$.

So $\sum_{n=1}^{\infty} \frac{1}{n!}$ converges by comparison.