Section 8.1: The Parabola as a Conic Section

Monday, March 10, 2014
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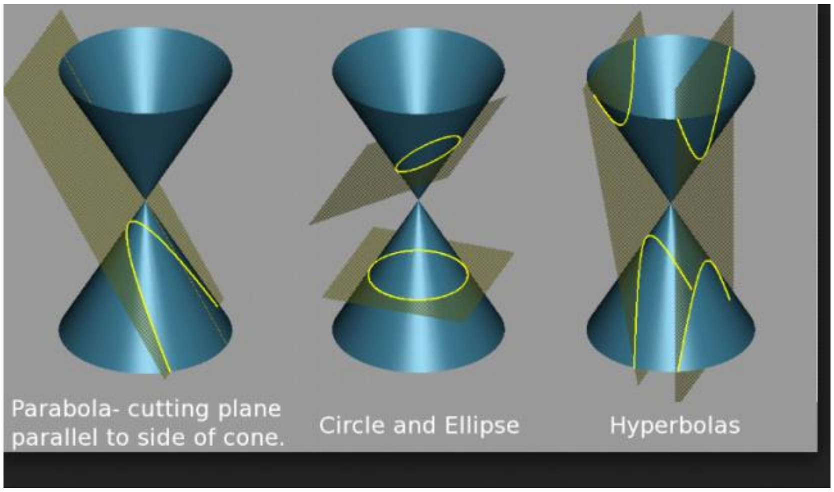
Goals:

1. Graph
2. Find vertex, focus, directrix
3. Find equation
4. apps



conic sections - Google Search
https://www.google.com/search?q=conic+sections&source=lnms&tbm=isch&sa=X&ei=M0weU9zsMYHloATbp4GoBw&sqi=2&ved=0CAcQ_AUoAQ&biw=1280&bih=833#facrc=_&imgdii=_&imgrc=wGRRaa0oYnRfUM%253A%3B0JfXrRA_RbVTsM%3Bhttp%253A%252F%252Fupload.wikimedia.org%252Fwikipedia%252Fcommons%252F4%252F48%252FConic_sections_2.png%3Bhttp%253A%252F%252Fen.wikipedia.org%252Fwiki%252FHyperbola%3B1080%3B600
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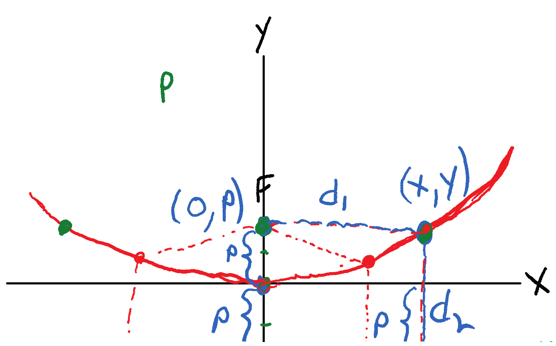
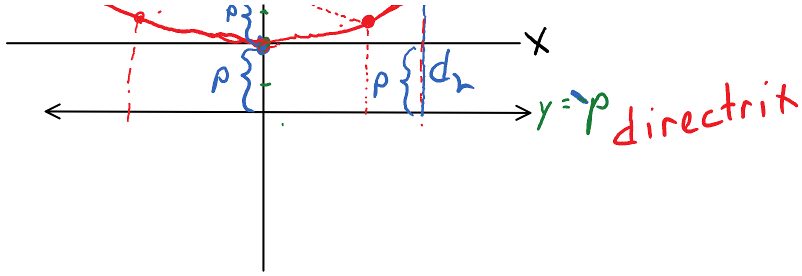
Def: A parabola is the set of all points equidistant from a line (directrix) and a point (focus).



$y = a(x-h)^2 + k$

$y = ax^2$



$$d_1 = d_2$$

$$d_1 = \sqrt{(x-0)^2 + (y-p)^2} = \sqrt{x^2 + (y-p)^2}$$

$$d_2 = y - (-p) = y + p$$

$$\underbrace{\sqrt{x^2 + (y-p)^2}}_{d_1} = \underbrace{y+p}_{d_2}$$

$$\left(\sqrt{x^2 + (y-p)^2}\right)^2 = (y+p)^2$$

$$x^2 + (y-p)^2 = (y+p)^2$$

$$x^2 + \cancel{y^2} - 2py + \cancel{p^2} = \cancel{y^2} + 2py + \cancel{p^2}$$

$$\begin{aligned} x^2 - 2py &= 2py \\ +2py \quad & \quad +2py \end{aligned}$$

$$\boxed{x^2 = 4py}$$

$\cot a = \frac{1}{4p}$

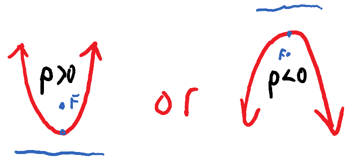
$$\boxed{x^2 = 4py}$$

$$y = \frac{1}{4p}x^2 \longrightarrow \text{set } a = \frac{1}{4p} \longrightarrow y = ax^2$$

$V(h,k)$

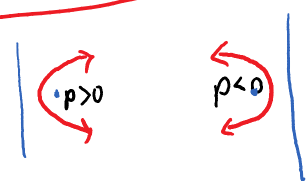
Standard forms of Parabola

★ $$\boxed{(x-h)^2 = 4p(y-k)}$$



Std form of a parabola that open vertically

★ $$\boxed{(y-k)^2 = 4p(x-h)}$$

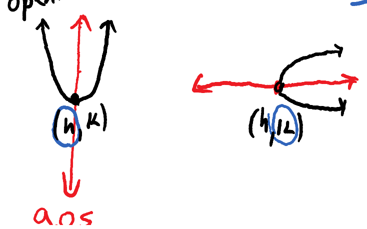


Std form opens horizontally

Notes

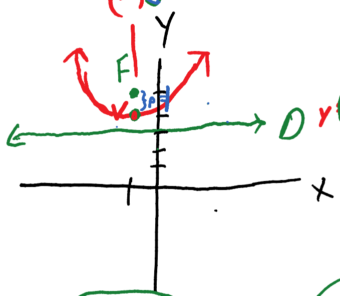
1. vertex is (h,k)
2. Axis of symmetry is $x = h$ or $y = k$

opens vertically   open horizontally



ex Find the eqn. of the parabola with $F(-1, 5)$ and directrix $y = 3$

height



$$(x - h)^2 = 4p(y - k)$$

-1 1

$$k = \frac{5+3}{2} = 4$$

$V(-1, 4)$

$$(x+1)^2 = 4(y-4)$$

$$p = 5 - 4 = 1$$

ex Find the vertex, focus, and directrix. Then graph.

a) $(x-3)^2 = -1(y+2)$

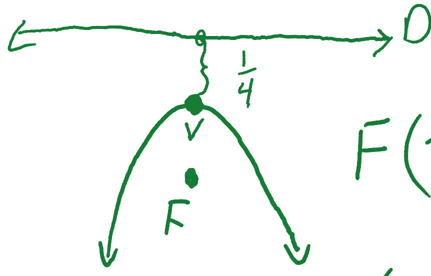
$$(x-h)^2 = 4p(y-k)$$

$V(3, -2)$

$4p = -1$

$p = -\frac{1}{4} < 0$

$F\left(3, -\frac{9}{4}\right)$



$k+p$

$F\left(3, -2 - \frac{1}{4}\right)$

$-\frac{8}{4}$

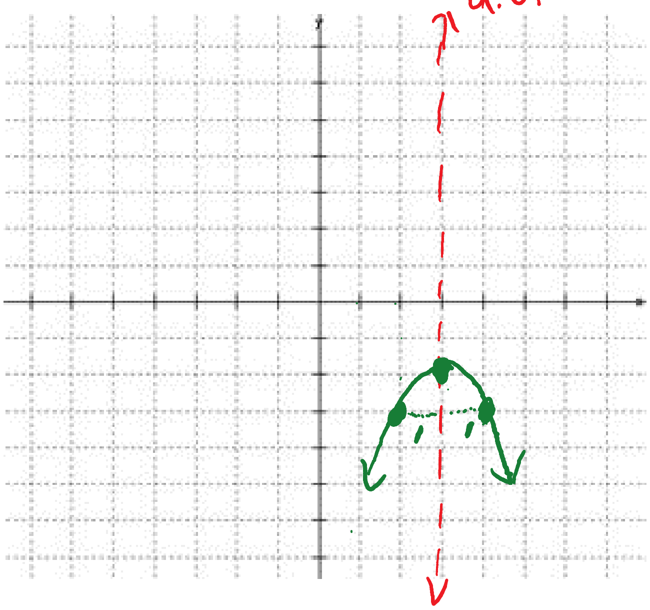
$F\left(3, -\frac{9}{4}\right)$

$y = k - p$

$D: \; y = -2 + \frac{1}{4}$

$D \quad y = -\frac{7}{4}$

a.o.s: $x = 3$



$(x-3)^2 = -(y+2)$

$V(3, -2)$

x	y
2	-3
4	-3

$1 = -(y+2)$

$-1 = y+2$

$-3 = y$

$3x - 4 = 0$

b) $2y^2 + 3x + 8y - 4 = 0$

$-3x \quad +4 \quad -3x+4$

$2y^2 + 7y = -3x + 4$

$2\left(y^2 + \frac{8}{2}y \quad\right) = -3x + 4$

$(y-k)^2 = 4p(x-h)$

$$2\left(y^2+\frac{8}{2}y \quad \right) = -3x+4$$

$$2\left(y^2+4y+2^2\right) = -3x+4+8$$

$+8$

$$2\left(y^2+4y+4\right) = -3x+4+8$$

$$\frac{\cancel{2}(y+2)^2}{\cancel{2}} = \frac{-3x+12}{2}$$

$$(y+2)^2 = \frac{-3\cdot(x-4)}{2}$$

$4p$

$$(y+2)^2 = \frac{-3}{2}(x-4)$$

$$-\frac{3}{\cancel{2}}\left(\frac{-2}{\cancel{-4}}\right) = 6$$

$$\sqrt{(y+2)^2} = \pm\sqrt{6}$$

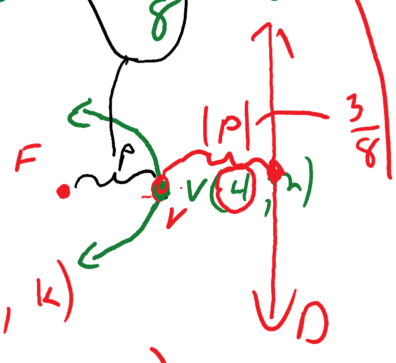
$$y+2 = \pm\sqrt{6}$$

$$y = -2\pm\sqrt{6}$$

$$V(4,-2)$$

$$\frac{1}{4}\cdot 4p = -\frac{3}{2}\cdot\frac{1}{4}$$

$$p = -\frac{3}{8} < 0$$



$$F(h+p,\ k)$$

$$F\left(\frac{4}{1}-\frac{3}{8},\ -2\right)$$

$$\frac{32}{8}-\frac{3}{8}$$

$$\frac{29}{8}$$

$$F\left(\frac{29}{8},\ -2\right)$$

$x = h - p$

8

$x = h - p$

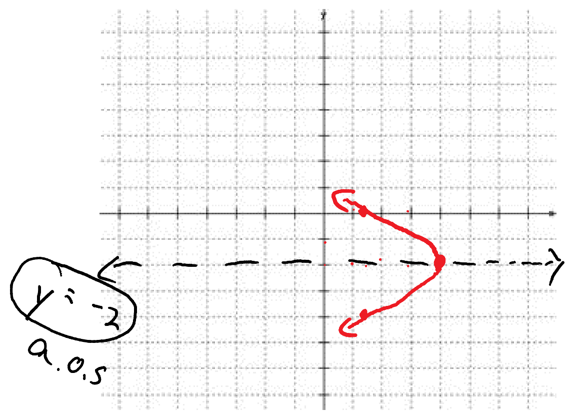
D: $x = 4 + \frac{3}{8}$

D: $x = \frac{35}{8}$

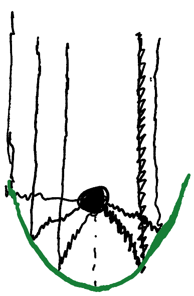
x	y
$\frac{4}{3}$	0
$\frac{4}{3}$	-4

$3x - 4 = 0$

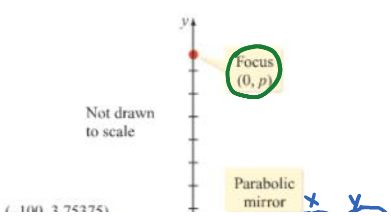
$x = \frac{4}{3} \approx 1.3$

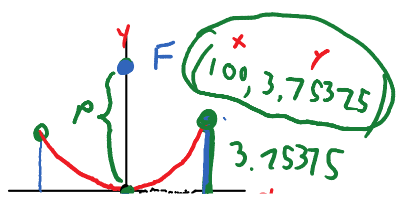


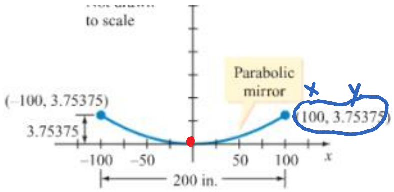
Reflective Property



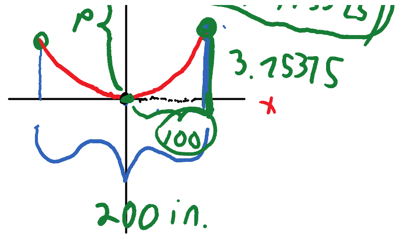
48. **The Hale Telescope** The parabolic mirror in the Hale Telescope at the Palomar Observatory in Southern California has a diameter of 200 inches and a concave depth of 3.75375 inches. Determine the location of its focus (to the nearest inch).







Cross section of the mirror in the Hale Telescope



Aufmann, College Algebra, 8e
http://www.webassign.net/ebooks/aufcolalg8/shell.html?s=0c0623bcf10fd69f750b5d894d9b23f1&c=249283&f=4721409
&type=youbook&id=462
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$$(x-h)^2 = 4p(y-k)$$

$$x^2 = 4py$$

$$\frac{100^2}{4(3.75375)} = \frac{\cancel{4}p\cancel{(3.75375)}}{\cancel{4(3.75375)}}$$

$$p = \frac{100^2}{(4(3.75375))}$$

$\boxed{p \approx 666 \text{ in}}$ above vertex

$$(0, 666)$$