Section 2: Immediate Inferences

Now that the basic structure of categorical propositions has been defined, we can begin to manipulate that structure. After all, what is the point of having basic building blocks if not to move them around? There are four ways to manipulate the basic structure of a categorical proposition *while making sure that it remains a categorical proposition.* These are:

1. Switch the order of the terms.

2. Replace a term with its complement.

3. Change the quality of the proposition.

4. Change the quantity of the proposition.

The fourth type of change was used in the Middle Ages (when there were only two recognized types of quantity), but we have no need of it, so we will just ignore it for now. Changes 1 – 3, when done correctly and in the right combinations, can produce alterations in the *form* of a sentence, while leaving the *meaning* the same. Such pairs of sentences may be called 'equivalent expressions', and the alteration that produces them is called an **immediate inference**. There are three immediate inferences: obversion, conversion, and contraposition. Let's consider the immediate inferences one at a time.

Obversion

We begin with obversion because it is the most important of the three immediate inferences. It is valid of *every* type of categorical proposition, so you will use it more often than the other two put together. Unlike conversion and contraposition, the terms don't move around. In fact the subject term remains entirely untouched. Only the predicate term changes, and it does not change its position. It is merely replaced by its complement. The other change in the proposition affects its *quality*. An affirmative proposition becomes negative, and a negative proposition becomes affirmative, at the same level of quantification.

> **Obversion** (valid for every categorical proposition) - change the quality of the statement, and replace the predicate term with its complement.
>
> [(Qual) S - P] *becomes...* [non-(Qual) S - non-P.]

Examples:

No A are B.	*becomes...*	All A are non-B.
All non-C are D.	*becomes...*	No non-C are non-D.
Most non-E are not non-F.	*becomes...*	Most non-E are F.
Some G are H.	*becomes...*	Some G are not non-H.

Obversion is the only immediate inference that is valid for all categorical propositions. It is valid for all of the intermediate propositions, including those that are not part of the common speech system. Obversion is the rule that captures the behavior of negatives in categorical propositions. It allows us to move the negative element of a proposition into the predicate term, and it allows us to transform the negative of a complementary term into a structural element of the proposition. It allows us to eliminate double negatives, or to create double negatives when it is convenient to have them. The validity of obversion is so obvious that students often ask, "Why must I have a special rule just to eliminate double negatives? Everybody knows that double negatives cancel. Why can't I just go ahead and do it without a rule?" In logic, every move must have a rule. The whole point of having rules is to explain what can and cannot be done. It is precisely because double negatives cancel that obversion is valid.

Exercises:
A. For each of the following, state the obversion.

1. No A are B.
2. Almost all non-C are D.
3. Most non-E are F.
4. Many G are not non-H.
5. Some non-I are non-J.
6. All K are non-L.
7. Few non-M are non-N.
8. All non-O are non-P.
9. Most Q are R.
10. Some S are non-T.

B. For each of the following, state the obversion.

1. Most red-colored fruit are not apples.
2. Some Americans are people uninterested in politics.
3. All doctors are concerned citizens.
4. Few senior citizens are persons without heart trouble.
5. All patriotic Americans are communist sympathizers.

Conversion

Conversion is the easiest of the three immediate inferences. Conversion simply reverses the order of the terms: the subject becomes the predicate, and the predicate becomes the subject.

> **Conversion** (valid only for E and I statements) - reverse the positions of the subject and predicate terms.
>
> [S - P] *becomes...* [P - S]

Examples:

No A are B.	*becomes...*	No B are A.
All non-C are D.	*becomes...*	All D are non-C.
Most non-E are non-F.	*becomes...*	Most non-F are non-E.
Some G are non-H.	*becomes...*	Some non-H are G.

It is fairly easy to see that conversion is valid for E and I statements. An E statement, 'No S are P', claims that there is no overlap between class S and class P. Obviously this relation (or rather lack of relation) is symmetrical. If there is no overlap between S and P, then there is also no overlap between P and S. An I statement, 'Some S are P', claims that there is some overlap between class S and class P. Again, this relation is obviously symmetrical. If S overlaps with P, then P also overlaps with S.

While conversion is valid for E and I statements, it is also important to see that conversion is *not* valid for any other type of categorical proposition. Consider, for example, the true A statement 'All dogs are mammals'. By conversion this would become 'All mammals are dogs', which is clearly false. Similarly, the true O statement, 'Some mammals are not dogs' becomes the false statement 'Some dogs are not mammals' by conversion. Conversion is not valid on any of the intermediate categorical propositions either, since the proportion that applies to the subject term may not apply to the predicate term. For example, 'Almost all S are P' may be true because S is a very small class, say 'Nobel Prize winners', while P is a relatively large class, say 'people over the age of 40'. It may be true that almost all Nobel Prize winners are people over the age of 40, but it is certainly not true that almost all people over the age of 40 are Nobel Prize winners. Weakening the quantifier to "most" and "many" shows that conversion is also not valid for the other intermediate propositions.

Exercises:

A. For each of the following, state the conversion.

1. No A are B.

2. Almost all non-C are D.

3. All non-E are non-F.

4. Most G are not H.

5. Some I are non-J.

B. For each of the following, state the conversion.

1. Some apples are not red-colored fruit.

2. Many Americans are people uninterested in politics.

3. Most doctors are concerned citizens.

4. Few underweight men are persons with heart trouble.

5. No communist sympathizers are patriotic Americans.

C. For all of the above exercises, state whether the requested transformation has been validly performed.

Contraposition

Contraposition is more involved than conversion. As in conversion, contraposition reverses the order of the terms. But it also requires that each term be replaced with its complement. Hence the complement of the subject becomes the predicate, while the complement of the predicate becomes the subject.

> **Contraposition** (valid only for A and O statements) - replace the subject and predicate terms with their respective complements, and reverse their positions.
>
> [S - P] *becomes...* [non-P - non-S]

Examples:

No A are B.	*becomes...*	No non-B are non-A.
All non-C are D.	*becomes...*	All non-D are C.
Most non-E are non-F.	*becomes...*	Most F are E.
Some G are not non-H.	*becomes...*	Some H are not non-G.

It is more difficult to see that contraposition is valid for A and O statements. It may be helpful to start with an example. If 'All dogs are mammals' is true, then 'All non-mammals are non-dogs' must also be true. Since all dogs are mammals, anything that *fails* to be a mammal (because it is a bird or a fish) must also *fail* to be a dog. Similarly, if 'Some mammals are not dogs' is true, then 'Some non-dogs are not non-mammals' must also be true. Cats, for example, are mammals; but cats are not dogs. Hence 'Some mammals are not dogs' is true. Likewise cats are

not non-mammals (which is to say that they are mammals), so 'Some non-dogs are not non-mammals' is also true. Contraposition captures the idea that if there is a relation between two terms, there must also be a relation between the complements of those terms. Contraposition is valid for A statements because the inclusion of class S into class P guarantees the inclusion of class non-P into class non-S. Contraposition is valid for O statements because the exclusion of some portion of class S from class P guarantees the exclusion of some portion of class non-P from class non-S.

However, while contraposition is valid for A and O statements, it is not valid for E statements, or for I statements, or for any of the intermediate propositions. For example, consider the true T statement 'Most flying animals are featherless animals'. (This statement is true, since most flying animals are insects, not birds). The contrapositive of this statement is 'Most feathered animals are non-flying animals', which is false. Most feathered animals do fly.

Exercises:
A. For each of the following, state the contraposition.

1. All A are B.

2. Few non-C are non-D.

3. Most non-E are F.

4. Many G are H.

5. Some non-I are not J.

B. For each of the following, state the contraposition.

1. Most red-colored fruit are not apples.

2. Some Americans are not people interested in politics.

3. All doctors are concerned citizens.

4. Few persons without heart trouble are senior citizens.

5. Many communist sympathizers are patriotic Americans.

C. For all of the above exercises, state whether the requested transformation has been validly performed.

Truth-Value

Any proposition is either true or false. Logicians call the truth or falsity of a proposition its **truth-value**.

A validly performed immediate inference is said to be **truth-value preserving**, because the resulting statement has the same truth-value as the statement on which the transformation was performed. On the other hand, if an immediate inference is invalidly performed, then the resulting statement may or may not have the same truth-value as the statement on which the transformation was performed. Since we cannot know whether the resulting statement is true or false, we will say that the truth-value of the resulting statement is *undetermined* by the truth-value of the original statement. It is important to understand that an invalid transformation does not reverse the truth value. That is, an invalid transformation does not make a true statement into a false one, or a false statement into a true one. An invalid transformation simply causes us to lose track of the truth-value altogether.

Examples:

If 'All A are B' is true, then 'All non-B are non-A' is true, since the second statement is a valid contraposition of the first.
If 'No C are D' is true, then 'No non-D are non-C' is undetermined, since the second statement is an invalid contraposition of the first.
If 'Some E are F' is false, then 'Some F are E' is false, since the second statement is a valid conversion of the first.
If 'Some G are not H' is false, then 'Some H are not G' is undetermined, since the second statement is an invalid conversion of the first.
If 'Most I are non-J' is true, then 'Most I are not J' is true, since the second statement is a valid obversion of the first.
If 'Most K are not L' is false, then 'Most K are non-L' is false, since the second statement is a valid obversion of the first.

Exercises:

A. In these exercises the second statement is derived from the first by a single transformation. State the kind of transformation, and state whether it was performed validly. Then fill in the blank with 'true', false', or 'undetermined'.

1. 'Most A are non-B' is true, so 'Most non-B are A' is_____.
   Kind of transformation_____; valid or invalid_____.

2. 'All C are non-D' is false, so 'All D are non-C' is_____.
   Kind of transformation_____; valid or invalid_____.

3. 'Some non-E are F' is true, so 'Some F are non-E' is_____.
   Kind of transformation_____; valid or invalid_____.

4. 'Many non-G are H' is false, so 'Many non-G are not non-H' is_____.
   Kind of transformation_____; valid or invalid_____.

5. 'Some I are not J' is true, so 'Some non-J are not non-I' is_____.
   Kind of transformation_____; valid or invalid_____.

6. 'Most non-K are not L' is false, so 'Most L are not non-K' is_____.
   Kind of transformation_____; valid or invalid_____.

7. 'Few M are non-N' is true, so 'Almost all M are N' is_____.
   Kind of transformation_____; valid or invalid_____.

8. 'No non-O are non-P' is false, so 'No non-P are non-O' is_____.
   Kind of transformation_____; valid or invalid_____.

9. 'Few non-Q are non-R' is true, so 'Few R are Q' is_____.
   Kind of transformation_____; valid or invalid_____.

10. 'Many S are T' is false, so 'Many T are S' is_____.
    Kind of transformation_____; valid or invalid_____.

B. In these exercises, state the kind of transformation that was performed, and state whether it was performed validly. Then fill in the blank with 'true', 'false', or 'undetermined'.

1. 'Some apples are not red-colored fruit' is true, so 'Some red-colored fruit are not apples' is_____.
   Kind of transformation_____; valid or invalid_____.

2 'Many Americans are people uninterested in politics' is false, so 'Many Americans are not people interested in politics' is_____.
   Kind of transformation_____; valid or invalid_____.

3. 'Most doctors are concerned citizens' is true, so 'Most unconcerned citizens are non-doctors' is_____.
   Kind of transformation_____; valid or invalid_____.

4. 'Few underweight men are persons with heart trouble' is false, so 'Almost all underweight men are persons without heart trouble' is_____.
   Kind of transformation_____; valid or invalid_____.

5. 'No communist sympathizers are patriotic Americans' is true, so 'No patriotic Americans are communist sympathizers' is_____.
   Kind of transformation_____; valid or invalid_____.

Two-step Transformations

It sometimes takes more than one transformation to show that two statements are equivalent expressions.

Example:

If 'Some non-A are not B' is true, then 'Some non-B are non-A' is true, since...

Some non-A are not B.

by obversion

Some non-A are non-B.

by conversion

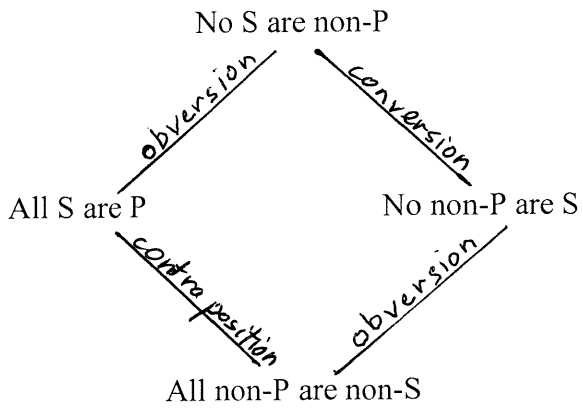
Some non-B are non-A.

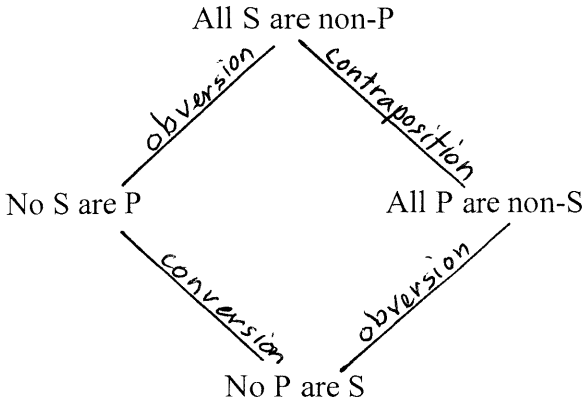
However, while it sometimes takes more than one transformation to show that two statements are equivalent, it never takes more than two. Since conversion and contraposition are not valid on the intermediate propositions, the only transformation available is obversion. A second obversion on the resulting statement merely takes us back to the first statement. Hence only one-step transformations are possible on intermediate propositions.

On the classical propositions we can do only slightly better. On A statements, for example, we have a choice between using obversion or contraposition. (Since conversion isn't valid on A statements, conversion isn't an option.) If we choose obversion, the resulting statement is an E statement. On E statements we have a choice between obversion and conversion. But obversion will simply take us back to the original A statement; so let us do the conversion. The result is a new E statement. Again we have a choice between obversion and conversion. Conversion will take us back to the E statement that we just left; so let us do the obversion. Now we arrive at a new A statement—the very same A statement, in fact, that we would have reached by doing contraposition in the first place. Contraposition on this statement takes us back to the beginning. Hence, on the classical propositions, two-step transformations are possible; but three-step transformations can be done more easily in one step, and four-step transformations are, likewise, a waste of time.

It is interesting to notice, by the way, that contraposition can be derived from conversion; and likewise, conversion can be derived from contraposition. That is, any statement that can be reached by contraposition can also be reached (more circuitously) by performing obversion-conversion-obversion. And similarly, any statement that can be reached by conversion can also be reached (more circuitously) by performing obversion-contraposition-obversion.

The following diagrams show how quickly conversion, contraposition, and obversion begin covering the same ground (so long as they are done *validly*).





Almost all S are P. ← obv. → Few S are non-P

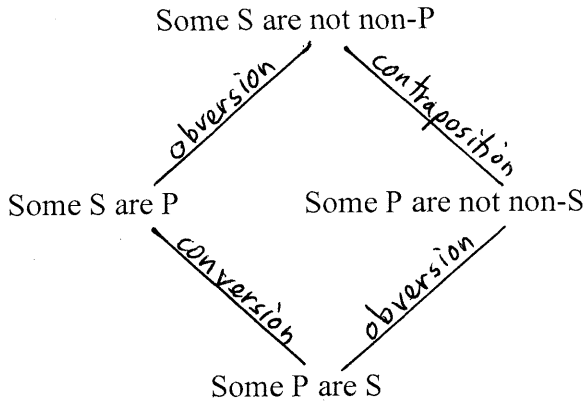
Few S are P ← obv. → Almost all S are non-P

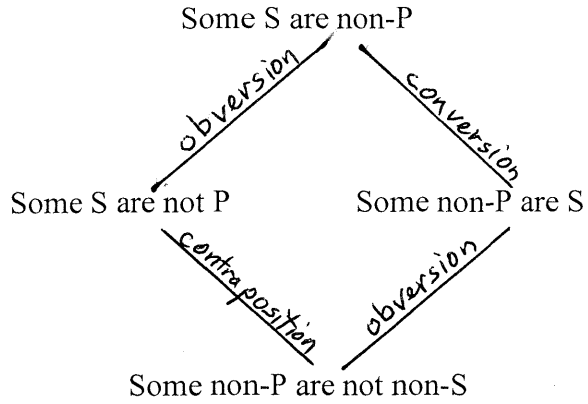
Most S are P ← obv. → Most S are not non-P

Most S are not P ← obv. → Most S are non-P

Many S are P ← obv. → Many S are not non-P

Many S are not P ← obv. → Many S are non-P





Exercises:

A. In these exercises the second statement may have been derived from the first using two valid transformations. Fill in the blank with 'true', false', or 'undetermined.

1. 'All A are B' is false, so 'All non-A are B' is_____.
2. 'No non-C are D' is true, so 'All D are C' is_____.
3. 'Most E are non-F' is false, so 'Most non-F are not non-E' is_____.
4. 'All non-G are non-H' is true, so 'No H are non-G' is_____.
5. 'Many non-I are not J' is false, so 'Many non-J are non-I' is_____.
6. 'All K are non-L' is true, so 'No L are K' is_____.
7. 'Some non-M are non-N' is false, so 'Some non-N are M' is_____.
8. 'Some O are non-P' is true, so 'Some non-P are not O' is_____.
9. 'Some Q are not R' is false, so 'Some non-R are Q' is_____.
10. 'Some non-S are not T' is true, so 'Some non-T are non-S' is_____.

B. Fill in the blank with 'true', false', or 'undetermined'.

1. 'Most red-colored fruit are not apples' is true, so 'Most non-apples are non-red-colored fruit' is_____.
2. 'Some Americans are not people interested in politics' is false, so 'Some people uninterested in politics are Americans' is_____.
3. 'All doctors are concerned citizens' is false, so 'All doctors are unconcerned citizens' is_____.
4. 'Few persons without heart trouble are senior citizens' is false, so 'Almost all senior citizens are persons with heart trouble' is_____.
5. 'Some patriotic Americans are communist sympathizers' is true, so 'Some communist sympathizers are not unpatriotic Americans' is_____.