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A Remark on Spelling

Observant readers are likely to notice that throughout this book I use the spelling ‘premiss’ rather than ‘premise’, despite the fact that the latter is the more common American spelling. In defense of the former spelling I offer the following, from Charles S. Peirce.

Since somebody may think that I write *premiss* instead of *premise* from negligence, may I be permitted to say that desperately negligent as I am of non-logical matters, I endeavor to attend to all the minutiae of logic. The word *praemissa* as a substantive meaning a premiss came into Latin very late and was never very common. Consequently, the English word was for a long time little used; but when it was used, it was always spelled *premiss*. But when it became more common so as to be written by persons of insufficient learning, it was confused with another word, the legal word, generally used in the plural, *premises*. This word is simply a French legal adjective meaning “aforesaid,” and commonly used in the phrase “les choses premises.” It thus passed into English in its plural form; and this plural form masked its adjectival nature, so that the unlearned did not know what it meant.¹

In English the word ‘premises’ has come to mean a piece of land, including the buildings and “appurtenances” that occupy it. The word is frequently used in such legalistic phrases as ‘occupy the premises’ and ‘vacate the premises’. I imagine that the custom on legal documents—eviction notices and the like—was to begin with a description of a certain property, and end with a phrase such as ‘The resident is hereby ordered to vacate the *aforesaid*’. With the French word used regularly in place of ‘aforesaid’, one would soon get the idea that ‘premises’ was a lawyers’ term of art meaning ‘real estate’. In any case, Peirce’s point (and mine) is that the word ‘premise’ has nothing to do with logic; and it is only by an unfortunate coincidence that the logical term ‘premiss’ has the same pronunciation.

Certain technical terms—notably the technical names for the quantification levels, ‘Particular’, ‘Common’, ‘Majority’, ‘Predominant’, and ‘Universal’—are also ordinary words in standard English. This fact occasionally creates problems. To avoid ambiguity I adopt the convention of capitalizing the first letter of these words when they are used in their technical sense. Hence, ‘a Particular proposition’ means a proposition that employs the quantifier ‘some’; while ‘a particular proposition’ means a given proposition (of unspecified type).

¹Charles S. Peirce, MS 75, an unpublished manuscript dated 1902.

INTRODUCTION

The Grammar of Argumentation

Few persons care to study logic, because everybody conceives himself to be proficient enough in the art of reasoning already. But I observe that this satisfaction is limited to one's own ratiocination, and does not extend to that of other men.¹ – Charles S. Peirce

As Peirce says, we all imagine that we are already sufficiently skilled in the art of reasoning, and therefore most of us see no reason to make a special study of the subject. And, to some extent, we are right. We begin learning how to think the moment we are born (perhaps even before), and by the time we have reached college age, we are quite accomplished at it. Indeed, anyone who can successfully find his shoes in the morning has achieved considerable expertise in the art of reasoning. Finding ones shoes requires all of the basic skills involved in critical thinking: generalizing from past experience, the formulating of reasonable hypotheses, the deduction of consequences that would follow from various hypotheses, and the testing of hypotheses through experimentation.

But, notice that there is a distinction between knowing how to reason and being able to explain how reasoning is done. In much the same way, there is a distinction between being able to digest food and being able to describe the chemical and biological processes involved in digestion. Understanding the chemistry of digestion will not make you better at digesting food. Likewise, knowing how to describe the structure of rational arguments will not necessarily make you a better thinker. A critical thinking class is unlikely to help you find your shoes. It will only help you to understand the process that you engage in whenever you set out to find your shoes.

On the other hand, it can't hurt. Knowing something about the chemistry of digestion may not improve the way in which enzymes interact with nutrients; but, it may make us better at selecting a healthy diet. Likewise, knowing how we reason may help us extrapolate our methods to more difficult and important problems than just the finding of shoes. It may be true that we begin learning how to think the moment we are born, but it does not follow that we have nothing further to learn. Reasoning, unlike digestion, is a skill, and it is always possible to improve. Indeed, as Peirce suggests, we may not be as good at reasoning as we think we are. The purpose of reasoning is to find the truth. If we were all as good at reasoning as we think we are, there would be a lot fewer differences of opinion in the world.

Learning how to describe the process of reasoning may be one way in which we can begin to improve our skill in reasoning, and so be better at finding the truth.

Terms, Propositions, and Arguments

I am sorry to have to tell you that the study of logic often feels a lot like the study of grammar; and, if few persons care to study logic, even fewer care to study grammar. However, it

cannot be helped. Thinking is done by means of what Peirce calls “signs,” by which he means anything that *refers* to something. We are chiefly interested in signs that can take the form of utterances, specifically, utterances that have “semantic content,” or meaning, such as words and groups of words. Grunts, moans, and disconnected syllables may also be considered utterances, but they do not *refer* in the way that words do, so they fall outside our study.

Utterances that have semantic content occur at three levels of complexity. Words, of course, have meaning in their own right, but we can also put words together to form more complex units of meaning called sentences. The rules by which words are put together to create sentences is what we mean by ‘grammar’. Hence, logic inevitably involves a certain amount of grammar. But, just as words can be put together to form sentences, so sentences can be put together to form a still more complex level of meaning called the argument. The rules by which sentences are put together to form arguments is what we mean by ‘logic’. In a sense, logic is the “grammar” of argumentation; although, since logic is the broader study, requiring consideration of all three levels of meaning, it is really more accurate to call grammar “the logic of sentences.”

Rather than talking about ‘words’ and ‘sentences’, logicians prefer to talk about ‘terms’ and ‘propositions’, as well as arguments. The distinction is subtle, but important. Some objects that we talk about are not individual *things* at all but are, rather, **ideal objects**. When we talk about ideal objects we are not talking about a mere physical manifestation, but about an “idea” which may be represented physically. For example, the letter ‘e’ is an ideal object. The letter ‘e’ occurs eight times in the previous sentence, twice in the word ‘example’, once in the word ‘the’, twice in ‘letter’, once by itself in quotation marks, and once each in the two words ‘ideal’ and ‘object’. But in another sense, there is only *one* letter ‘e’, namely the fifth letter of the alphabet. An ideal object is the unity behind all of its various manifestations. Here are some examples.

TERMS

- | | | |
|-----|--|---|
| (a) | ‘this particular piece of paper.’ | <i>Although there are two different phrases here, nevertheless, only one idea is being expressed. Hence there is only one term.</i> |
| (b) | ‘the piece of paper on which this is written.’ | |

PROPOSITIONS

- | | | |
|------|---|--|
| (a’) | ‘This piece of paper is white.’ | <i>Although there are two different sentences here, nevertheless, each expresses the same combination of ideas. Hence there is only one proposition.</i> |
| (b’) | ‘The piece of paper on which this is written is white.’ | |

ARGUMENTS

- | | | |
|------|---|--|
| (a’) | ‘This particular piece of paper is white. Hence some paper is white.’ | <i>Although these two passages are worded differently, each one offers the same evidence in support of the same claim. Hence there is only one argument.</i> |
| (b’) | ‘Some paper is white, since this specific piece of paper is white.’ | |

While words and sentences, like letters, are ideal objects, they are more concrete than terms and propositions. That is, two words, such as “cats” and “felines,” must be regarded as different individual words; yet, each expresses the same term, since the words are synonyms. Likewise, different individual sentences may be used to express identically the same proposition, and by extension, different individual sentences may be used to construct identically the same argument. Since there is often more than one way to express the same idea, you should avoid relying upon identical words and phrases in your analyses of arguments. The exercises in this book tend to make things easy, by using precisely the same wording every time the same term is expressed. Real authors of arguments do just the reverse. Since using different words and phrases to express the same idea tends to make an argument more interesting to read, real authors tend to use different wording even when they have the same idea in mind. That is one reason that arguments encountered in “real life” are usually more difficult to analyze than those encountered in logic textbooks. (In fact re-wording for the sake of variety is not always a good idea. A certain amount of repetition will often make your writing clearer, and give it a feeling of unity.)

Complex Terms and Degenerate Arguments

Terms, propositions, and arguments differ from each other both with respect to their structure and with respect to their function.

Definitions by *function*:

A **term** presents a topic for discussion, but says nothing about the topic so identified.

A **proposition** urges some fact upon the listener, but offers no justification of the fact so urged.

An **argument** manifests the truth of one fact by connecting it to another fact that is presumably already taken to be true.

Definitions by *basic structure*:

A **term** is a simple, or unanalyzed, utterance. (Being complex sequences of sounds, they are obviously open to *some* sort of analysis, but that analysis falls outside the scope of logic.)

A **proposition** is a pair of terms, called the **subject** and the **predicate**, related in such a way that the topic referred to by the predicate is also referred to by the subject, to a greater or lesser extent.

An **argument** is a pair of propositions, called the **premiss** and the **conclusion**, related in such a way that the fact urged by the conclusion is also urged by the premiss, to a greater or lesser extent.

EXAMPLES:

<p><i>Simple Term:</i></p> <p>‘cats’</p>
<p><i>Simple Proposition:</i></p> <p>‘Cats are animals.’</p>
<p><i>Simple Argument:</i></p> <p>‘Cats are animals; so some animals are cats.’</p>

Now the picture becomes complex. To some extent it is possible to mix and match structure with function, using an utterance that has the structure of a proposition to perform the function of a term, or using an utterance that has the structure of an argument to perform the function of a proposition. But this is possible only to some extent. Structure places some limits

on possible functions. Consider the following image. It is possible to break a window by throwing a rock through it, it is possible to break a window by throwing a typewriter through it, and it is even possible to break a window by throwing a laptop computer through it. Any of these three instruments will serve, since breaking a window is not a very complex activity. It is, of course, also possible to use a typewriter to write a letter, but it is not possible to use a rock to write a letter. Writing a letter is a more complex activity, which requires a more complex or highly structured instrument. You may use a typewriter to do a rock's job, but you cannot use a rock to do a typewriter's job. Likewise, you can use a laptop computer to type a letter (assuming you can send the document to a printer), but you can also use a laptop computer to check your email, make a spreadsheet, go online, etc. A typewriter can do none of those things. Again, you can use a computer to do a typewriter's job, but you cannot use a typewriter to do a computer's job. And, of course, you can use a computer to do a rock's job, but you cannot use a rock to do a computer's job.

Similarly, functioning as an argument is a very complex activity, and it can only be done by highly structured units of discourse. By contrast, functioning as a term is a very simple activity, and it can be done by pretty much any unit of discourse at all. Propositions fall somewhere in the middle.

The situation is described by the following matrix:

	TERM	PROPOSITION	ARGUMENT
	<i>(as viewed according to function or use in context)</i>		
TERM	<i>Simple term:</i> 'cats'		
PROPOSITION	<i>Complex term (Degenerate proposition):</i> 'black cats'	<i>Simple proposition:</i> 'Some cats are black.'	
ARGUMENT <i>(as viewed according to structure)</i>	<i>Complex term (Degenerate argument):</i> 'a cat's being bad luck because of its being black'	<i>Complex proposition (Degenerate argument):</i> 'If some cats are black, then they are bad luck.'	<i>Simple argument:</i> 'Black cats are bad luck. Some cats are black, so they are bad luck.'

A simple utterance, such as 'cats' or 'black', can present a topic for discussion. But such an utterance cannot be used to urge a fact upon a listener. It simply does not have a sufficiently

complex structure to permit it to *assert* something. Likewise, a pair of terms related by predication can urge a fact upon a listener, but it cannot appeal to the connectedness of one fact to another in order to defend a further fact. Again this is because it does not have a sufficiently complex structure.

But going the other direction, there is no reason why a related pair of terms related by predication should not be used merely to present a topic for discussion. For example, the utterance 'black cats' is a pair of terms related by predication, i.e. it predicates 'black' of 'cats'. But the utterance is not being used to *urge the fact* that some cats are black. It merely offers black cats to us as a topic for further discussion. An utterance that has the structure of a proposition, but is used as if it were a mere term, may be regarded as a 'degenerate proposition', although for our purposes, it is more useful to call such degenerate propositions 'complex terms'.

Similarly, a group of propositions related by an inference has a sufficiently complex structure that it could be used for all three purposes. Besides genuine arguments, there will naturally be two types of "degenerate" arguments. The first will be utterances having the structure of an argument that merely urge a fact upon the listener, and the second will be utterances having the structure of an argument that merely present a topic for discussion.

There are two important practical consequences to all of this:

- (1) Terms (meaning utterances that have the function of terms) can be any size whatsoever. A term is not restricted to a single word, but may involve complex phrases. Moreover, notice that terms are not restricted to naming concrete objects, but may refer to highly abstract concepts as well.
- (2) Some propositions have the structure of arguments, and may, therefore, be mistaken for arguments, if one is not careful.

Deduction, Induction, and Retrodution

The ancient Greeks were the first to examine the basic structures of rational argumentation. Aristotle laid out the basic structure of argumentation in a book called *Prior Analytics*. Aristotle's description of reasoning was accepted through the middle ages as pretty much the final word on the subject. However, Aristotle's description of argumentation really describes only one kind of argument: the kind that we call deduction. At the end of the middle ages, Francis Bacon realized that the reasoning used by the scientists of his time were not well described by Aristotle's deductive syllogisms. Bacon proposed, in a book called *Novum Organum*, that a new kind of logic, induction, was as important as deduction. At the end of the 19th Century, Charles S. Peirce further expanded our understanding of the basic patterns of reasoning by proposing that there are actually three patterns of rational thought: deduction, induction, and a third pattern that he (eventually) called retrodution.

Peirce's belief that there are three types of reasoning was based, in part at least, on the idea that, using syllogisms to represent forms of argumentation, there are precisely three logically possible forms that argumentation can take. To use Peirce's example, suppose we discover a room in which there are several bags of beans of various colors. Suppose we also notice a handful of beans lying on a table. We may draw various conclusions, depending upon what we know about the situation.

Deduction—In a deduction, we begin with a RULE, or general statement, and a CASE, which is a class or instance subsumed under this rule. The conclusion is an OBSERVATION of fact, which Peirce called the “result.” (I prefer the term “observation,” since it more accurately describes the characteristics of this statement in all three types of argumentation.)

RULE - All the beans in this bag are white.
CASE - The beans on the table are from this bag.
OBSERVATION - The beans on the table are white.

In a deductive argument, the conclusion follows from the premisses because of the meanings of the connector words used to express the argument—words, such as ‘if’, ‘and’, ‘not’, etc. Connector words do not, by themselves, make any reference to objects; they merely structure propositions (and arguments). Thus, they have meaning only when they occur *with* content-bearing words. In the above example, the argument turns primarily on the meanings of the words ‘all’ and ‘are’, which express the concept of class inclusion. Given the agreed meaning of these words, the premisses in effect *stipulate* that the conclusion is true. Thus, in a deductive argument the conclusion cannot be false unless at least one of the premisses is false. This makes deduction the most powerful type of reasoning, but also the least applicable to our experiences.

Induction—In an induction, we begin with an OBSERVATION made upon a sample, or CASE. The conclusion is the generalization, or RULE.

OBSERVATION - The beans on the table are white.
CASE - The beans on the table are from this bag.
RULE - All the beans in this bag are white.

The persuasive power of an inductive argument comes from the fact that the sample is taken as representative of the larger population from which it was drawn. Thus, some care must be taken to *cause* the sample to reflect the population, such as drawing the sample in a random manner, or otherwise making sure that distinct elements of the population are represented in the sample. Errors are, of course, possible, so the conclusion may be false, even if the premisses are both true. The premisses do not stipulate that the conclusion is true, but they do *indicate* that the conclusion is true. Induction is not as powerful as deduction, but it is, in many ways, more important, since it applies directly to our experiences with the world around us.

Retroduction—In a retroduction, we also begin with an OBSERVATION, usually a surprising fact that catches our attention and demands an explanation. The explanation is suggested by a RULE, or general statement, that has something in common with what we have observed. The conclusion is, then, the CASE which explains the surprising observation by fitting it into a general pattern.

RULE - All the beans in this bag are white.
OBSERVATION - The beans on the table are white.
CASE - The beans on the table are from this bag.

A retroductive argument follows because of some sort of similarity, or “concomitance.” Any type of concomitance may be used, including (but not limited to) location in space and time, similarity of size, color, material composition, etc. The concomitance could easily turn out to be just an irrelevant coincidence, so retroduction, like induction, may have true premisses and a false conclusion.² Indeed, retroduction is the weakest of the three types of reasoning. It cannot even indicate that its conclusion is likely to be true, it can only *suggest* that its conclusion *might* be true. Fortunately human beings have an uncanny instinct for being able to recognize the difference between relevant connections and irrelevant ones. Retroduction may be the weakest form of reasoning, but it is nevertheless important. Deduction and induction both merely draw out the consequences of given premisses, so no new knowledge emerges from them. Only retroduction has the ability to suggest *new* ideas, since—psychologically speaking—the RULE is decided upon at the same time that the conclusion is proposed. Retroduction is the kind of reasoning involved in discovery and invention. We could not get along without it.

Validity and Critique

I have offered a rather unromantic view of logic according to which logic is closely akin to grammar. The business of grammar is to distinguish between well-formed, or grammatical, sentences and ill-formed, or ungrammatical sentences. Similarly, the business of logic is to distinguish between well-formed and ill-formed *arguments*. A well-formed argument is called a **valid** argument; while an ill-formed argument—an argument that does not follow correct form—is called an **invalid** argument.

However, determining whether an argument is valid or invalid has little to do with the *critique* of the argument. That is, the purpose of validity is not to tell whether an argument is good or bad. Rather, the purpose of validity is to tell us which type of reasoning the argument employs. I have worked out criteria for the validity of syllogisms under which any deductively valid argument will also meet the criteria for inductive and retroductive validity. Any inductively valid argument will also meet the criteria for retroductive validity. Hence, an argument that meets the criteria for deductive validity may be said to be a deductive argument; an argument that fails the criteria for deductive validity, but meets the criteria for inductive validity, may be called an inductive argument; and, of course, an argument that fails to be either deductive or inductive may still be a valid retroductive argument.³ An “argument” that fails to meet *any* criteria of validity is not, strictly speaking, an argument at all. Invalid arguments exist only in the sense in which ungrammatical sentences exist. There are, of course, ungrammatical sentences: student papers are often full of them. But such groupings of words are called “sentences” only by courtesy. In the same way, an invalid “argument” is an argument only by courtesy, in recognition of the author’s (unsuccessful) intentions.

Once it has been determined what type of validity an argument possesses, it then remains to be decided whether an argument is good or bad, i.e. whether or not we should permit ourselves to be persuaded by it. In the case of deduction, this is relatively easy: we only need to consider whether or not the premisses are true. With regard to induction and retroduction the critique is more difficult. It is not enough merely to be convinced that the premisses are true; other criteria must also be considered. In any case, only in the case of deduction does the truth of the premisses *force* or *compel* us to accept the truth of the conclusion. The true premisses of an induction *indicate* that a conclusion is true, but indications—evidence—can be affected by extraneous forces. A weathervane might point east, leading us to infer that the wind is blowing out of the west, although in fact the wind is out of the north. Our conclusion is false. However, the

assertion, “The weathervane is pointing east,” remains true, even after we discover that the weathervane is rusted in place and *always* point east. So it may be wise to trust the conclusions presented to us by inductive evidence, but we should never feel *compelled* to accept them.

Nearly all logic or critical thinking texts refer to what are called “formal fallacies,” that is, to forms of reasoning that fail to be deductively valid. One form is called Affirming the Consequent, for example, “If it rains there will be clouds in the sky. Look! There are clouds in the sky, so it will rain.” In fact, the so-called “formal fallacies” are *valid forms*, but they are valid forms of retroductive reasoning, not deductive or inductive reasoning. The error—or fallacy—in this example is not in its form, but in our failure to recognize the force with which the conclusion should be put forward. Mere clouds hardly *force* us to believe that it will rain. They even fall short of providing *evidence* that it will rain. But they do *suggest the possibility* of rain. This is a valid retroductive argument. When I go out, I may (reasonably) decide to take an umbrella with me, just in case.

Notes

1. Charles S. Peirce, “The Fixation of Belief,” *Philosophical Writings*, ed. Justus Buchler, Dover Publications, New York, 1955.

2. This is not, however, enough to establish that retroduction *is* induction, unless one defines induction as any argument in which the premisses could be true while the conclusion is false. But, by this definition, any unrelated pair of statements designated “premiss” and “conclusion” would be an inductive argument. In short, this definition confuses “induction” with the concept of deductive “invalidity.” In fact, just as retroduction shares certain characteristics with induction that neither shares with deduction, retroduction shares certain characteristics with deduction that neither shares with induction. Finally, of course, deduction and induction share certain characteristics with each other that neither shares with retroduction.

3. What it is that makes the form of an argument “correct” for its type is discussed in B. E. R. Thompson, “Deductively valid, inductively valid, and retroductively valid syllogisms,” *Transactions of the Charles S. Peirce Society*, Vol 54, No 4 (2016). Pp. 611 – 632. However, my more recent work on the validity of non-deductive syllogisms is currently unpublished.