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Chapter 8 ROOTS, RADICALS, AND ROOT FUNCTIONS

8.3 Simplifying Radical Expressions

Learning Objectives
1 Use the product rule for radicals.
2 Use the quotient rule for radicals.
3 Simplify radicals.
4 Simplify products and quotients of radicals with different indexes.
5 Use the Pythagorean theorem.
6 Use the distance formula.

Key Terms

Use the vocabulary terms listed below to complete each statement in exercises 1−3.

index **radicand** **hypotenuse** **legs**

1. In a right triangle, the side opposite the right angle is called the
_________________________.

2. In the expression $\sqrt[4]{x^2}$, the "4" is the _________________________ and x^2 is the
_________________________________.

3. In a right triangle, the sides that form the right angle are called the
_______________________.

Objective 1 Use the product rule for radicals.

Video Examples

Review these examples for Objective 1:

1. Multiply. Assume that all variables represent positive real numbers.

a. $\sqrt{13} \cdot \sqrt{5}$

$\sqrt{13} \cdot \sqrt{5} = \sqrt{13 \cdot 5} = \sqrt{65}$

c. $\sqrt{5x} \cdot \sqrt{2yz}$

$\sqrt{5x} \cdot \sqrt{2yz} = \sqrt{10xyz}$

Now Try:

1. Multiply. Assume that all variables represent positive real numbers.

a. $\sqrt{2} \cdot \sqrt{7}$

____

c. $\sqrt{3} \cdot \sqrt{11mn}$

____

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2. Multiply. Assume that all variables represent positive real numbers.

a. $\sqrt[4]{2} \cdot \sqrt[4]{2x}$

$\sqrt[4]{2} \cdot \sqrt[4]{2x} = \sqrt[4]{2 \cdot 2x} = \sqrt[4]{4x}$

b. $\sqrt[3]{8x} \cdot \sqrt[3]{2y^2}$

$\sqrt[3]{8x} \cdot \sqrt[3]{2y^2} = \sqrt[3]{8x \cdot 2y^2} = \sqrt[3]{16xy^2}$

c. $\sqrt[5]{6r^2} \cdot \sqrt[5]{4r^2}$

$\sqrt[5]{6r^2} \cdot \sqrt[5]{4r^2} = \sqrt[5]{6r^2 \cdot 4r^2} = \sqrt[5]{24r^4}$

d. $\sqrt[5]{2} \cdot \sqrt[4]{6}$

$\sqrt[5]{2} \cdot \sqrt[4]{6}$ cannot be simplified using the product rule for radicals, because the indexes (5 and 4) are different.

2. Multiply. Assume that all variables represent positive real numbers.

a. $\sqrt[3]{3} \cdot \sqrt[3]{7}$ ____________

b. $\sqrt[3]{7x} \cdot \sqrt[3]{5y}$ ____________

c. $\sqrt[5]{4w} \cdot \sqrt[5]{2w^3}$ ____________

d. $\sqrt{3} \cdot \sqrt[3]{64}$ ____________

Objective 1 Practice Exercises

For extra help, see Examples 1–2 on page 499 of your text.

Multiply. Assume that variables represent positive real numbers.

1. $\sqrt{7x} \cdot \sqrt{6t}$ **1.** ____________

2. $\sqrt[5]{6r^2t^3} \cdot \sqrt[5]{4r^2t}$ **2.** ____________

3. $\sqrt{3} \cdot \sqrt[3]{7}$ **3.** ____________

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Objective 2 Use the quotient rule for radicals.

Video Examples

Review these examples for Objective 2:

3. Simplify. Assume that all variables represent positive real numbers.

a. $\sqrt{\frac{64}{9}}$

$$\sqrt{\frac{64}{9}} = \frac{\sqrt{64}}{\sqrt{9}} = \frac{8}{3}$$

b. $\sqrt{\frac{5}{16}}$

$$\sqrt{\frac{5}{16}} = \frac{\sqrt{5}}{\sqrt{16}} = \frac{\sqrt{5}}{4}$$

c. $\sqrt[3]{-\frac{27}{8}}$

$$\sqrt[3]{-\frac{27}{8}} = \frac{\sqrt[3]{-27}}{\sqrt[3]{8}} = \frac{-3}{2} = -\frac{3}{2}$$

e. $\sqrt[5]{-\frac{a^3}{243}}$

$$\sqrt[5]{-\frac{a^3}{243}} = \frac{\sqrt[5]{a^3}}{\sqrt[5]{-243}} = \frac{\sqrt[5]{a^3}}{-3} = -\frac{\sqrt[5]{a^3}}{3}$$

f. $\sqrt{\frac{z^4}{36}}$

$$\sqrt{\frac{z^4}{36}} = \frac{\sqrt{z^4}}{\sqrt{36}} = \frac{z^2}{6}$$

Now Try:

3. Simplify. Assume that all variables represent positive real numbers.

a. $\sqrt{\frac{36}{49}}$ ____________

b. $\sqrt{\frac{13}{81}}$ ____________

c. $\sqrt[3]{-\frac{343}{125}}$ ____________

e. $\sqrt[3]{-\frac{a^6}{125}}$ ____________

f. $\sqrt[4]{\frac{m}{81}}$ ____________

Objective 2 Practice Exercises

For extra help, see Example 3 on page 500 of your text.

Simplify each radical. Assume that variables represent positive real numbers.

4. $\sqrt[3]{\frac{27}{8}}$

4. ____________

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5. $\sqrt[5]{\dfrac{7x}{32}}$

5. ________________

6. $\sqrt[3]{-\dfrac{a^6}{125}}$

6. ________________

Objective 3 Simplify radicals.

Video Examples

Review these examples for Objective 3:

4. Simplify.

a. $\sqrt{90}$

$$\begin{aligned}\sqrt{90} &= \sqrt{9 \cdot 10}\\ &= \sqrt{9} \cdot \sqrt{10}\\ &= 3\sqrt{10}\end{aligned}$$

b. $\sqrt{288}$

$$\begin{aligned}\sqrt{288} &= \sqrt{144 \cdot 2}\\ &= \sqrt{144} \cdot \sqrt{2}\\ &= 12\sqrt{2}\end{aligned}$$

c. $\sqrt{35}$

No perfect square (other than 1) divides into 35, so $\sqrt{35}$ cannot be simplified further.

d. $\sqrt[3]{81}$

$$\sqrt[3]{81} = \sqrt[3]{27 \cdot 3} = \sqrt[3]{27} \cdot \sqrt[3]{3} = 3\sqrt[3]{3}$$

e. $-\sqrt[4]{3125}$

$$\begin{aligned}-\sqrt[4]{3125} &= -\sqrt[4]{5^5} = \sqrt[4]{5^4 \cdot 5}\\ &= -\sqrt[4]{5^4} \cdot \sqrt[4]{5}\\ &= -5\sqrt[4]{5}\end{aligned}$$

Now Try:

4. Simplify.

a. $\sqrt{84}$

____

b. $\sqrt{162}$

____

c. $\sqrt{95}$

____

d. $\sqrt[3]{256}$

____

e. $-\sqrt[5]{512}$

____

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5. Simplify. Assume that all variables represent positive real numbers.

a. $\sqrt{81x^3}$

$\sqrt{81x^3} = \sqrt{9^2 \cdot x^2 \cdot x} = 9x\sqrt{x}$

b. $\sqrt{56x^7y^6}$

$$\begin{aligned}\sqrt{56x^7y^6} &= \sqrt{4\cdot 14\cdot \left(x^3\right)^2\cdot x\cdot \left(y^3\right)^2} \\ &= 2x^3y^3\sqrt{14x}\end{aligned}$$

c. $\sqrt[3]{-270b^4c^8}$

$$\begin{aligned}\sqrt[3]{-270b^4c^8} &= \sqrt[3]{\left(-27b^3c^6\right)\left(10bc^2\right)} \\ &= \sqrt[3]{-27b^3c^6}\cdot\sqrt[3]{10bc^2} \\ &= -3bc^2\sqrt[3]{10bc^2}\end{aligned}$$

d. $-\sqrt[6]{448a^7b^7}$

$$\begin{aligned}-\sqrt[6]{448a^7b^7} &= -\sqrt[6]{(64a^6b^6)(7ab)} \\ &= -\sqrt[6]{64a^6b^6}\cdot\sqrt[6]{7ab} \\ &= -2ab\sqrt[6]{7ab}\end{aligned}$$

6. Simplify. Assume that all variables represent positive real numbers.

a. $\sqrt[24]{5^4}$

$\sqrt[24]{5^4} = (5^4)^{1/24} = 5^{4/24} = 5^{1/6} = \sqrt[6]{5}$

c. $\sqrt[8]{x^{12}}$

$$\begin{aligned}\sqrt[8]{x^{12}} &= (x^{12})^{1/8} = x^{12/8} = x^{3/2} = \sqrt{x^3} \\ &= \sqrt{x^2\cdot x} = \sqrt{x^2}\cdot\sqrt{x} = x\sqrt{x}\end{aligned}$$

5. Simplify. Assume that all variables represent positive real numbers.

a. $\sqrt{100y^3}$

____

b. $\sqrt{48m^5r^9}$

____

c. $\sqrt[3]{-32n^7t^5}$

____

d. $-\sqrt[4]{405x^3y^9}$

____

6. Simplify. Assume that all variables represent positive real numbers.

a. $\sqrt[12]{11^9}$

____

c. $\sqrt[24]{z^{30}}$

____

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Objective 3 Practice Exercises

For extra help, see Examples 4–6 on pages 501–502 of your text.

Simplify each radical. Assume that variables represent positive real numbers.

7. $\sqrt[42]{x^{28}}$ **7.** ________________

8. $\sqrt{8x^3y^6z^{11}}$ **8.** ________________

9. $\sqrt[3]{1250a^5b^7}$ **9.** ________________

Objective 4 Simplify products and quotients of radicals with different indexes.

Video Examples

Review this example for Objective 4:

7. Simplify $\sqrt{3}\cdot\sqrt[5]{6}$.

Because the different indexes, 2 and 5, have least common multiple index of 10, we use rational exponents to write each radical as a tenth root.

$$\sqrt{3}=3^{1/2}=3^{5/10}=\sqrt[10]{3^5}=\sqrt[10]{243}$$

$$\sqrt[5]{6}=6^{1/5}=6^{2/10}=\sqrt[10]{6^2}=\sqrt[10]{36}$$

$$\sqrt{3}\cdot\sqrt[5]{6}=\sqrt[10]{243}\cdot\sqrt[10]{36}$$

$$=\sqrt[10]{243\cdot 36}$$

$$=\sqrt[10]{8748}$$

Now Try:

7. Simplify $\sqrt[3]{3}\cdot\sqrt[6]{7}$.

____

Objective 4 Practice Exercises

For extra help, see Example 7 on page 503 of your text.

Simplify each radical. Assume that variables represent positive real numbers.

10. $\sqrt{r}\cdot\sqrt[3]{r}$ **10.** ________________

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11. $\sqrt[4]{2} \cdot \sqrt[8]{7}$

11. ________________

12. $\sqrt{3} \cdot \sqrt[5]{64}$

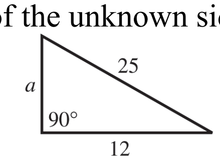
12. ________________

Objective 5 Use the Pythagorean theorem.

Video Examples

Review this example for Objective 5:

8. Use the Pythagorean theorem to find the length of the unknown side of the triangle.

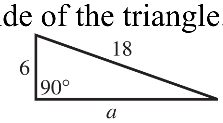


$$a^2 + b^2 = c^2$$
$$12^2 + b^2 = 25^2$$
$$144 + b^2 = 625$$
$$b^2 = 481$$
$$b = \sqrt{481}$$

The length of the side is $\sqrt{481}$.

Now Try:

8. Use the Pythagorean theorem to find the length of the unknown side of the triangle.



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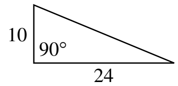
Objective 5 Practice Exercises

For extra help, see Example 8 on page 504 of your text.

Find the unknown length in each right triangle. Simplify the answer if necessary.

13.



13. ________________

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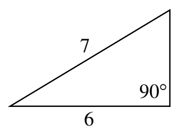
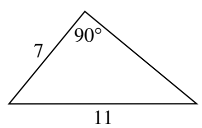
14.



14. ________________

15.



15. ________________

Objective 6 Use the distance formula.

Video Examples

Review this example for Objective 6:

9. Find the distance between the points (2,–2) and (–6, 1).

Use the distance formula. Let $(x_1, y_1) = (2, -2)$ and $(x_2, y_2) = (-6, 1)$.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-6-2)^2 + [1-(-2)]^2} \\ &= \sqrt{(-8)^2 + 3^2} \\ &= \sqrt{64+9} \\ &= \sqrt{73} \end{aligned}$$

Now Try:

9. Find the distance between the points (–1,–2) and (–4, 3).

____

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Objective 6 Practice Exercises

For extra help, see Example 9 on page 505 of your text.

Find the distance between each pair of points.

16. $(3, 4)$ and $(-1,-2)$ **16.** ________________

17. $(-2,-3)$ and $(-5, 1)$ **17.** ________________

18. $(4, 2)$ and $(3,-1)$ **18.** ________________

