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Chapter 10 NONLINEAR FUNCTIONS, CONIC SECTIONS, AND NONLINEAR SYSTEMS

10.4 Nonlinear Systems of Equations

Learning Objectives
1. Solve a nonlinear system using substitution.
2. Solve a nonlinear system with two second-degree equations using elimination.
3. Solve a nonlinear system that requires a combination of methods.

Key Terms

Use the vocabulary terms listed below to complete each statement in exercises 1−2.

nonlinear equation **nonlinear system of equations**

1. An equation in which some terms have more than one variable or a variable of degree 2 or greater is a ______________________________.

2. A system with at least one nonlinear equation is a ____________________.

Objective 1 Solve a nonlinear system using substitution.

Review these examples for Objective 1:

1. Solve the system.

$$x^2+3y^2=3 \quad (1)$$
$$x+y=-1 \quad (2)$$

The graph of (1) is an ellipse and the graph of (2) is a line, so the graphs could intersect in zero, one, or two points.

Solve (2) for x, then substitute that expression into (1) and solve for y.

$$x+y=-1 \quad (2)$$
$$x=-y-1$$
$$x^2+3y^2=3 \quad (1)$$
$$(-y-1)^2+3y^2=3$$
$$y^2+2y+1+3y^2=3$$
$$4y^2+2y-2=0$$
$$2(2y-1)(y+1)=0$$
$$(2y-1)(y+1)=0$$
$$2y-1=0 \quad \text{or} \quad y+1=0$$
$$y=\tfrac{1}{2} \qquad\qquad y=-1$$

Now Try:

1. Solve the system.

$$x^2+y^2=17$$
$$x+y=-3$$

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To solve for x, substitute $y=\frac{1}{2}$ in (2): $x=-\frac{3}{2}$.
To solve for x, substitute $y=-1$ in (1): $x=0$.
The solution set is $\left\{\left(-\frac{3}{2}, \frac{1}{2}\right), (0, -1)\right\}$.

2. Solve the system.

$$xy=-10 \quad (1)$$
$$2x-y=9 \quad (2)$$

The graph of (1) is a hyperbola and the graph of (2) is a line. There may be zero, one, or two points of intersection. Since neither equation has a squared term, solve either equation for one of the variables and then substitute the result into the other equation.

Solving (1) for y gives $y=-\frac{10}{x}$. Substituting into (2) gives

$$2x-\left(-\frac{10}{x}\right)=9$$
$$2x+\frac{10}{x}=9$$
$$x\left[2x+\frac{10}{x}\right]=9x$$
$$2x^2+10=9x$$
$$2x^2-9x+10=0$$
$$(2x-5)(x-2)=0$$
$$2x-5=0 \quad \text{or} \quad x-2=0$$
$$x=\frac{5}{2} \qquad\qquad x=2$$

To solve for y, substitute $x=\frac{5}{2}$ in (1): $y=-4$.
To solve for y, substitute $x=2$ in (1): $y=-5$.
The solution set is $\left\{(2, -5), \left(\frac{5}{2}, -4\right)\right\}$.

2. Solve the system.

$$xy=1$$
$$x+y=2$$

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Objective 1 Practice Exercises

For extra help, see Examples 1–2 on pages 702–704 of your text.

Solve each system by the substitution method.

1. $2x^2-y^2=-1$
$2x+y=7$

1. ______________

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2. $xy = -6$
$x + y = 1$

2. ________________

3. $xy = 24$
$y = 2x + 2$

3. ________________

Objective 2 Solve a nonlinear system with two second-degree equations using elimination.

Review this example for Objective 2:

3. Solve the system.

$$3x^2 + y^2 = 35 \quad (1)$$
$$2x^2 - y^2 = 15 \quad (2)$$

The graph of (1) is an ellipse and the graph of (2) is a hyperbola. There may be zero, one, or two points of intersection. Adding the two equations will eliminate y.

$$\begin{aligned} 3x^2 + y^2 &= 35 \quad (1) \\ \underline{2x^2 - y^2} &\underline{= 15} \quad (2) \\ 5x^2 &= 50 \\ x^2 &= 10 \\ x &= \pm\sqrt{10} \end{aligned}$$

Substitute the values for x in (1) and solve for y.

For $x = \sqrt{10}$,
$$\begin{aligned} 3(\sqrt{10})^2 + y^2 &= 35 \\ 30 + y^2 &= 35 \\ y^2 &= 5 \\ y &= \pm\sqrt{5} \end{aligned}$$

For $x = -\sqrt{10}$,
$$\begin{aligned} 3(-\sqrt{10})^2 + y^2 &= 35 \\ 30 + y^2 &= 35 \\ y^2 &= 5 \\ y &= \pm\sqrt{5} \end{aligned}$$

The solution set is $\{(\sqrt{10}, -\sqrt{5}), (\sqrt{10}, \sqrt{5}), (-\sqrt{10}, \sqrt{5}), (-\sqrt{10}, -\sqrt{5})\}$.

Now Try:

3. Solve the system.

$$x^2 - 2y = 8$$
$$x^2 + y^2 = 16$$

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Objective 2 Practice Exercises

For extra help, see Example 3 on pages 704–705 of your text.

Solve each system by the elimination method.

4. $5x^2 - y^2 = 55$
$2x^2 + y^2 = 57$

4. ________________

5. $x^2 + 2y^2 = 11$
$2x^2 - y^2 = 17$

5. ________________

6. $3x^2 + 2y^2 = 30$
$2x^2 + y^2 = 17$

6. ________________

Objective 3 Solve a nonlinear system that requires a combination of methods.

Review this example for Objective 3:

4. Solve the system.

$$x^2 + 5xy - y^2 = 13 \quad (1)$$
$$x^2 - \quad y^2 = 3 \quad (2)$$

We will use the elimination method in combination with the substitution method. Multiply Eq (2) by –1 and add to Eq (1).

$$\begin{array}{rl} x^2 + 5xy - y^2 = 13 & (1) \\ \underline{-x^2 + \quad y^2 = -3} & (2) \\ 5xy = 10 & \\ y = \frac{2}{x} & (3) \end{array}$$

Now Try:

4. Solve the system.

$$5x^2 - xy + 5y^2 = 89$$
$$x^2 + \quad y^2 = 17$$

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Now substitute $\frac{2}{x}$ for y in (2) and solve for x.

$$x^2-\left(\frac{2}{x}\right)^2=3$$
$$x^2-\frac{4}{x^2}=3$$
$$x^4-4=3x^2$$
$$x^4-3x^2-4=0$$
$$(x-2)(x+2)(x^2+1)=0$$

Using the zero-factor property, we have
$x=2$, $x=-2$, $x=i$, $x=-i$.
Substitute each of these values into (3) and solve for y.
If $x=2$, then $y=1$.
If $x=-2$, then $y=-1$.
If $x=i$, then $y=\frac{2}{i}=\frac{2}{i}\cdot\frac{-i}{-i}=\frac{-2i}{-i^2}=-2i$.
If $x=-i$, then $y=\frac{2}{-i}=\frac{2}{-i}\cdot\frac{i}{i}=\frac{2i}{-i^2}=2i$.
It is important to check all answers in the original equations because it is possible to obtain extraneous solutions. The solution set is $\{(2, 1), (-2, -1), (i, -2i), (-i, 2i)\}$.

Objective 3 Practice Exercises

For extra help, see Example 4 on pages 705–706 of your text.

Solve each system.

7. $4x^2-2xy+4y^2=64$
$x^2+y^2=13$

7. ________________

8. $x^2+3xy+2y^2=12$
$-x^2+8xy-2y^2=10$

8. ________________

9. $x^2+5xy-y^2=20$
$x^2-2xy-y^2=-8$

9. ________________

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Chapter 10 NONLINEAR FUNCTIONS, CONIC SECTIONS, AND NONLINEAR SYSTEMS

10.5 Second-Degree Inequalities and Systems of Inequalities

Learning Objectives
1 Graph second-degree inequalities.
2 Graph the solution set of a system of inequalities.

Key Terms

Use the vocabulary terms listed below to complete each statement in exercises 1−2.

second-degree inequality **system of inequalities**

1. A ________________________________ consists of two or more inequalities to be solved at the same time.

2. A(n)____ is an inequality with at least one variable of degree 2 and no variable with degree greater than 2.

Objective 1 Graph second-degree inequalities.

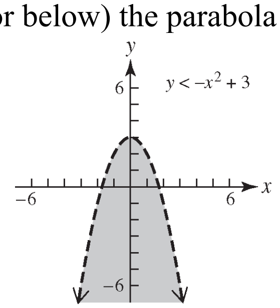
Review these examples for Objective 1:

2. Graph $y < -x^2 + 3$.

The boundary, $y = -x^2 + 3$, is a parabola that opens down with vertex (0, 3).
Use (0, 0) as a test point.

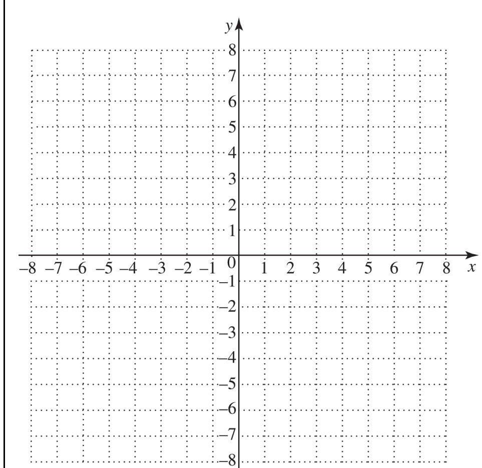
$$y < -x^2 + 3$$
$$0 \overset{?}{<} -0^2 + 3$$
$$0 < 3 \quad \text{True}$$

Because the final inequality is a true statement, the points in the region containing (0, 0) satisfy the inequality. The parabola is drawn as a dashed curve since the points on the parabola itself do not satisfy the inequality and the region inside (or below) the parabola is shaded.



Now Try:

2. Graph $y \geq x^2 - 4$.



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3. Graph $16x^2 < 9y^2 + 144$.

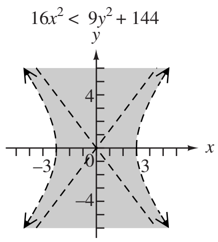
Rewrite the inequality.

$16x^2 - 9y^2 < 144$

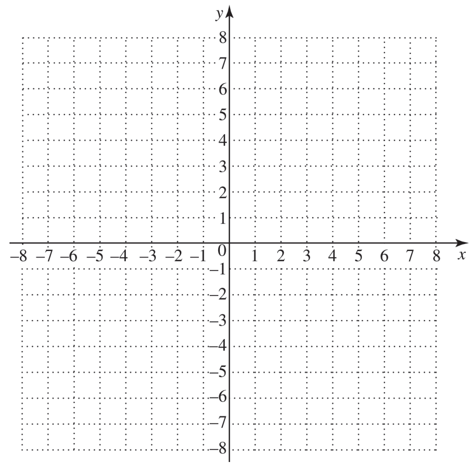
$\frac{x^2}{9} - \frac{y^2}{16} < 1$

The boundary, drawn as a dashed curve, is the following hyperbola. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Since the graph is a horizontal hyperbola, the desired region will be either between the branches or the regions to the right of the right branch and to the left of the left branch. Using the test point (0, 0) into the original inequality leads to 0 < 1, a true statement. So the region between the branches containing (0, 0) is shaded.



3. Graph $9y^2 - 36x^2 > 144$.



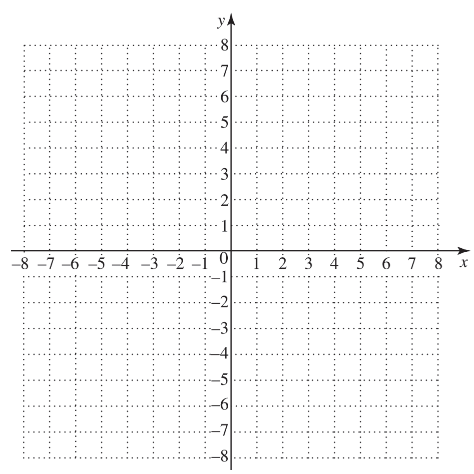
Objective 1 Practice Exercises

For extra help, see Examples 1–3 on pages 709–710 of your text.

Graph each inequality.

1. $x \le 2y^2 + 8y + 9$

1.



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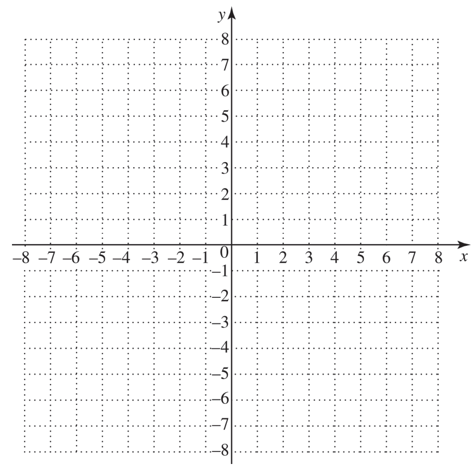
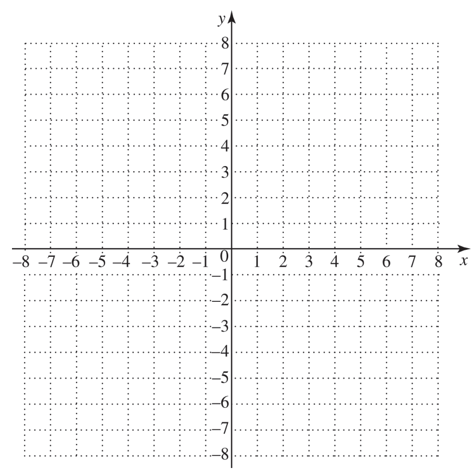
2. $x^2 + 9y^2 > 36$

2.



3. $9x^2 - y^2 < 36$

3.



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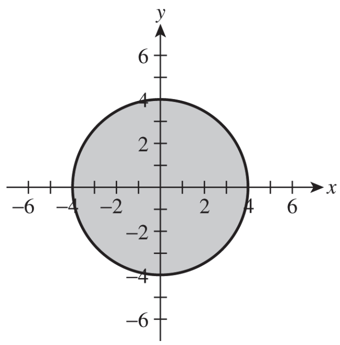
Objective 2 Graph the solution set of a system of inequalities.

Review these examples for Objective 2:

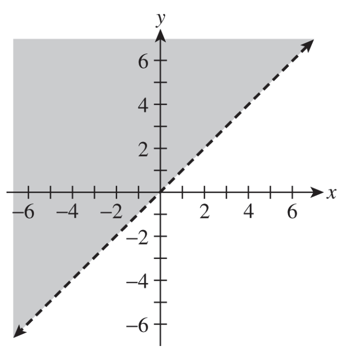
5. Graph the solution set of the system.

$$x^2 + y^2 \le 16$$
$$y > x$$

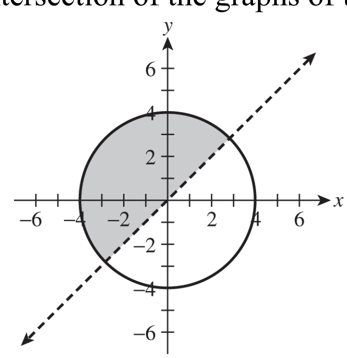
Begin by graphing $x^2 + y^2 \le 16$. The boundary line is a circle centered at the origin with radius 4. The test point (0, 0) leads to a true statement, so we shade inside the circle.



The boundary of the solution set of $y > x$ is a dashed line passing through (0, 0). Using the test point (0, 1) leads to a true statement, so shade above the line.



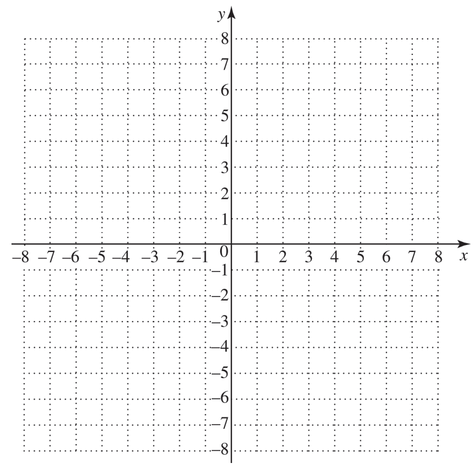
The graph of the solution set of the system is the intersection of the graphs of the two inequalities.



Now Try:

5. Graph the solution set of the system.

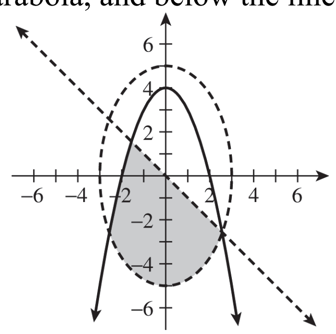
$$4y + x^2 < 0$$
$$x \ge 0$$



7. Graph the solution set of the system.

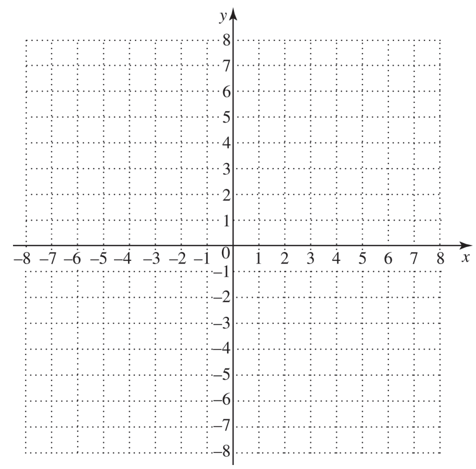
$$25x^2 + 9y^2 < 225$$
$$y \le -x^2 + 4$$
$$y < -x$$

The graph of $25x^2 + 9y^2 < 225$ is a dashed ellipse with $a = 3$ and $y = 5$. To satisfy the inequality, a point must lie inside the ellipse. The graph of $y \le -x^2 + 4$ is a parabola with vertex (0, 4) opening downward. The inequality includes the points on the boundary along with the points inside the parabola. The graph of $y < -x$ includes all points below the line $y = -x$. Therefore, the graph of the system is the shaded region, which lies inside the ellipse and the parabola, and below the line.



7. Graph the solution set of the system.

$$y \ge (x+2)^2 - 5$$
$$2x + y < -5$$
$$(x+3)^2 + (y-1)^2 < 9$$



Objective 2 Practice Exercises

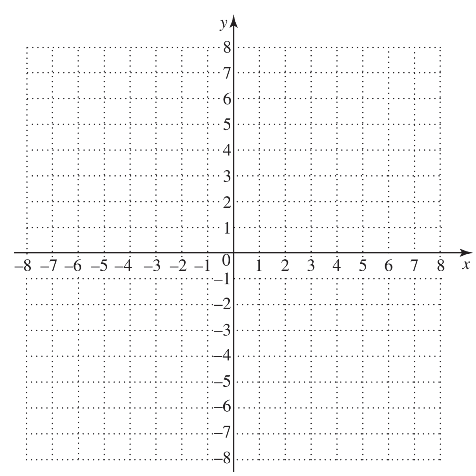
For extra help, see Examples 4–7 on pages 710–712 of your text.

Graph each system of inequalities.

4. $-x + y > 2$

$3x + y > 6$

4.



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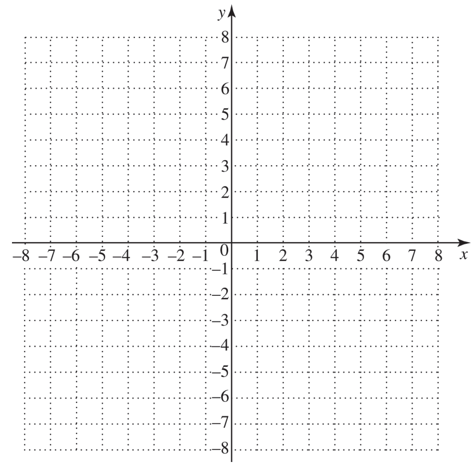
5. $x^2 + y^2 \leq 25$

$3x - 5y > -15$

5.

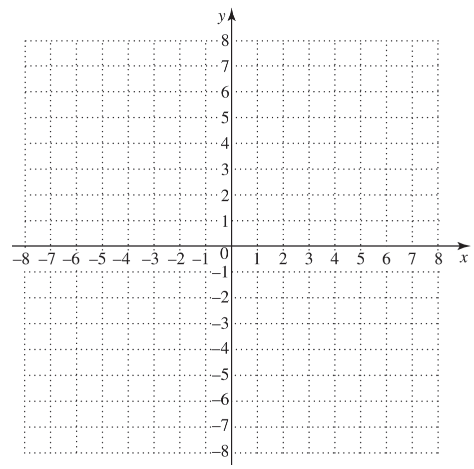


6. $x^2 + 4y^2 \leq 36$

$-5 < x < 2$

$y \geq 0$

6.



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