SAMPLE TEST: PRECALCULUS (130 / 135 / 140)

Topic I - RATIONAL EXPRESSIONS

1. Divide and simplify $(x-\frac{1}{x})\div(1-\frac{1}{x})$

2. Perform the indicated operations and simplify $\dfrac{\frac{2}{x}-1}{x^2-4}$

3. Simplify $\dfrac{5u^2}{v^3}\dfrac{-4v}{uv}(3v^2)$

4. Subtract and simplify $\dfrac{10}{x^2-4}-\dfrac{3-x}{x^2+2x}$

5. Add and Simplify $\dfrac{1}{(a-b)(a+2b)}+\dfrac{1}{(a+2b)(a-3b)}+\dfrac{1}{(a-3b)(a-b)}$

6. Perform the indicated operations and simplify $(\frac{x}{y}-\frac{y}{x})^{-1}$

7. Divide and simplify $\dfrac{\frac{a^2b}{a-b}}{\frac{a+b}{a^2-b^2}}$

8. Perform the indicated operations and simplify $\dfrac{\frac{x}{x-3}-\frac{2x}{x^2-2x-3}}{\frac{2}{x+1}-\frac{1}{x}}$

Topic II - EXPONENTS AND RADICALS

1. Evaluate $32^{-3/5}$

2. Simplify $\sqrt{10}\sqrt{15}$

3. Simplify $\sqrt{(u^2-v^2)(u+v)}, u>v>0$

4. Simplify $\sqrt{16x^2+36y^2}$

5. Simplify $\dfrac{(5ab^2)(2a^3b)^2}{a^3b^2}$

6. Simplify $\dfrac{x^{-2}y^3z}{x^{-1}y^{-2}z^0}$

7. Simplify $\dfrac{x^{u^2-3}}{x^{u-3}}$

8. Simpllify $(x^{a-1})^{a+1}$

9. Rationalize $\dfrac{a}{\sqrt[3]{4}}$

10. Simplify $\dfrac{9^4 x^{3/2} y^{5/2}}{81^6 x^{-1/2} y^{-1}}$

11. Simplify $(-27x^{12}y^{-18})^{-2/3}$

12. Simplify $\sqrt[4]{a^6} - 2a\sqrt{a}$

13. Simplify and express without radicals $\dfrac{\sqrt{x}}{\sqrt[11]{x}}$

14. Simplify $\sqrt{18} - 3\sqrt{8} + \sqrt{50}$

Topic III - LINEAR EQUATIONS, INEQUALITIES, AND ABSOLUTE VALUE

1. Solvle for x: $x = \dfrac{m}{n}x + 2$

2. Solve for a: $\sqrt{a-1} - 2 = 2$

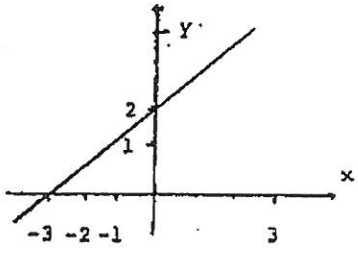
3. Solve for x: $\dfrac{2}{x} - \dfrac{2}{2x+1} = \dfrac{1}{x+1}$

4. Solve the system of equations for x and y:
$$3x + 5y = -7$$
$$4x - 2y = 8$$

5. Solve the system of equations for x and y:
$$5x = 3y$$
$$3x + 2y = 21$$

6. If a = -2 then evaluate: $|a-1| - |-a| + 1$

7. Find the equation of the line in the figure shown.



8. Solve for x: $|4-3x| \le 7$

Topic IV - POLYNOMIALS AND POLYNOMIAL EQUATIONS

1. Solve for x: $3x + 5x^{1/2} - 28 = 0$

2. Solve for x: $-2x^2 + 2x + 1 = 0$

3. Solve for x: $(x-1)(x-2) = 1$

4. Solve for x by completing the square: $4x^2 + 3x - 1 = 0$

5. Solve for x: $\sqrt{2x-5} - \sqrt{x-2} = 2$

6. Divide $x^3 - 10x + 3$ by $x+3$

7. Verify that 2 is a root of the polynomial $6x^3 - 19x^2 + 9x + 10$ and factor this polynomial completely.

8. Find all values of a so that the polynomial $ax^2 + 5x + 2$ has two distinct real roots.

9. Solve for x: $x^2 + 2x - 3 > 0$

10. Graph the equation: $y = x^2 - 2x + 3$

Topic V - FUNCTIONS

1. $f(u) = \dfrac{-3u^2 + 2u + 3}{au^2 + bu + 1}$. Find $f(0)$

2. $f(x) = -\dfrac{3}{x-4}$. Find $f\left(\dfrac{1}{x+2}\right)$ and simplify.

3. $f(x) = 3x + 1$, $g(x) = x^2 - 1$. Find $f(g(x))$ and $g(f(x))$.

4. If $f(x) = \dfrac{2x+1}{x-1}$, for which x does $f(x) = -5$?

5. $f(t)=\dfrac{2t-1}{t+3}$. Find $\dfrac{5}{f(3)}$.

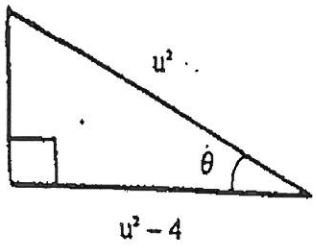
6. Graph the function $f(x)=\dfrac{1}{x-3}$

7. Graph the function $h(x)=|x^2-1|$

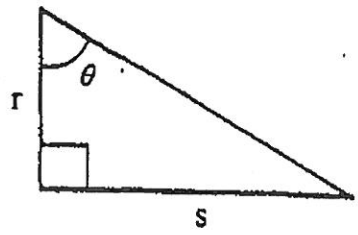
8. Find the domain and range of the function $g(x)=\sqrt{10+2x-x^2}$

Topic VI - TRIGONOMETRY

1. Given θ as shown in the figure to the right, find $\sin\theta$



2. Given θ as shown in the figure to the right, find $\tan\theta\cos\theta$



3. Verify the identity: $\tan(\pi-x)=-\tan x$

4. Verify the identity: $\sec y - \cos y = \tan y \sin y$

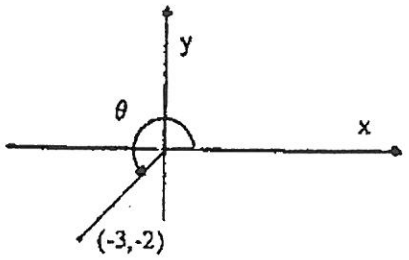
5. Verify the identity: $\tan x + \tan y = \dfrac{\sin(x+y)}{\cos x \cos y}$

6. Find all x satisfying: $\tan^2 2x = 3,\ 0 \le x \le \pi$

7. Graph the equation: $y = \cos 3x$

8. Graph the equation: $y = 3\sin\dfrac{x}{2}$

9. In the figure shown to the right find sec θ.



10. If $0 < \theta < 2\pi$ find all values of θ for which $2\sin\theta = \tan\theta$

11. Express 108° in radians.

12. $\cos\dfrac{5\pi}{6} =$

Topic VII - LOGARITHMIC AND EXPONENTIAL FUNCTIONS

1. Solve for x: $\log_a x = b$

2. Solve for u: $3^u = 4$

3. Evaluate $\log_{49}\sqrt{7}$

4. Evaluate $\log_4\left(\dfrac{4^{-9}}{32}\right)$

5. Which of the following is larger? $5-\log_2 60,\ \ 3-\log_2 20$

6. Simplify the following: $\log x^2 y-\frac{1}{2}\log x+3\log y$

7. Graph the equation: $y=(\frac{1}{2})^x$

8. Graph the equation: $y=|\log_{10} x|$

9. Solve for x: $5^{3x}-1=0$

10. Solve for x:` $\log_{10}(3x+1)=3$

Topic VIII - WORD PROBLEMS

1. Sue is 2 years older than John. 15 years ago she was twice as old as he was. If x and y are the ages of Sue and John now, give a system of equations that could be solved to find x and y.

2. A 3x5 photograph is enlarged so that its width measures 7". What is the length of the enlargement?

3. If the circumference of a circle is multiplied by 5, how much is the area increased?

4. Two numbers add to 17, and 7 times the first minus five times the second is 3. What are the numbers?

5. A positive number is taken to the 1/3 power and the result is squared. The final answer is 9. What is the original number?

6. The sine of twice an angle is $\frac{\sqrt{2}}{2}$. If the angle is between 0 and 2π what are its possible values?

7. The price of a plane ticket has been increased by 15% to $172.50. What was the cost before the increase?

8. The radius of a circle is increased by 20%. What is the percent increase in area?

Solutions for Sample Test: Precalculus

Topic I - RATIONAL EXPRESSIONS

1. $x+1$
2. $\dfrac{-1}{x^2+2x}$
3. $\dfrac{-60u}{v}$
4. $\dfrac{x+3}{x(x-2)}$
5. $\dfrac{3a-2b}{(a-b)(a+2b)(a-3b)}$
6. $\dfrac{xy}{x^2-y^2}$
7. a^2b
8. $\dfrac{x^2}{x-3}$

Topic II - EXPONENTS AND RADICALS

1. $\dfrac{1}{8}$
2. $5\sqrt{6}$
3. $(u+v)\sqrt{u-v}$
4. $2\sqrt{4x^2+9y^2}$
5. $20a^4b^2$
6. $\dfrac{y^5z}{x}$
7. x^{u^2-u}
8. x^{a^2-1}
9. $\dfrac{a(\sqrt[3]{2})}{2}$
10. $\dfrac{x^2y^{7/2}}{9^8}$
11. $\dfrac{y^{12}}{9x^8}$
12. $-a\sqrt{a}$
13. $x^{9/22}$
14. $2\sqrt{2}$

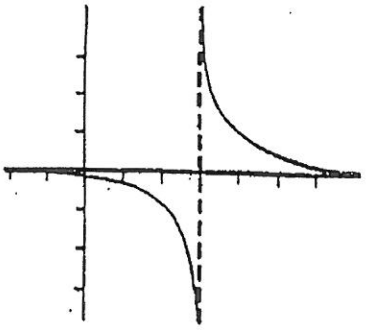
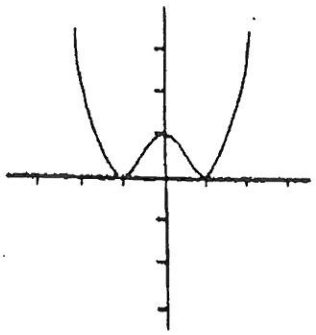
Topic III - LINEAR EQUATIONS, INEQUALITIES AND ABSOLUTE VALUE

1. $\dfrac{2n}{n-m}$
2. $a=17$
3. $x=-\dfrac{2}{3}$
4. $x=1,\ y=-2$
5. $x=\dfrac{63}{19},\ y=\dfrac{105}{19}$
6. 2
7. $2x-3y=-6$ or $y=\frac{2}{3}x+2$
8. $-1\le x\le\dfrac{11}{3}$

Topic IV - POLYNOMIALS AND POLYNOMIAL EQUATIONS

1. $x = \frac{49}{9}$
2. $x = \frac{1 \pm \sqrt{3}}{2}$
3. $x = \frac{3 \pm \sqrt{5}}{2}$
4. $x = -1,\ x = \frac{1}{4}$
5. $x = 27$
6. $x^2 - 3x - 1 + \frac{6}{x+3}$
7. $(x-2)(2x+1)(3x-5)$
8. $a < \frac{25}{8}$
9. $x < -3$ or $x > 1$
10. 

Topic V - FUNCTIONS

1. $f(0) = 3$
2. $\frac{3x+6}{4x+7}$
3. $f(g(x)) = 3x^2 - 2$
   $g(f(x)) = 9x^2 + 6x$
4. $x = \frac{4}{7}$
5. 6
6. 
7. 

8. Domain: $1-\sqrt{11} \le x \le 1+\sqrt{11}$ Range: $0 \le y \le \sqrt{11}$

Topic VI -TRIGONOMETRY

1. $\sin\theta = \dfrac{2\sqrt{2u^2-4}}{u^2}$

2. $\dfrac{s}{\sqrt{r^2+s^2}}$

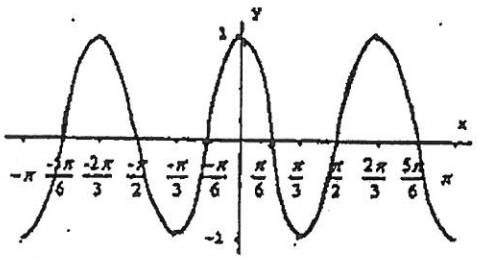
3. $\tan(\pi - x) = \dfrac{\sin(\pi - x)}{\cos(\pi - x)} = \dfrac{\sin x}{-\cos x} = -\tan x$

4. $\sec y - \cos y = \dfrac{1}{\cos y} - \cos y = \dfrac{1-\cos^2 y}{\cos y} = \dfrac{\sin^2 y}{\cos y} = \tan y \ \sin y$

5. $\tan x + \tan y = \dfrac{\sin x}{\cos x} + \dfrac{\sin y}{\cos y} = \dfrac{\sin x \ \cos y + \sin y \ \cos x}{\cos x \ \cos y} = \dfrac{\sin(x+y)}{\cos x \ \cos y}$

6. $x = \dfrac{\pi}{6}, \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{5\pi}{6}$

7.



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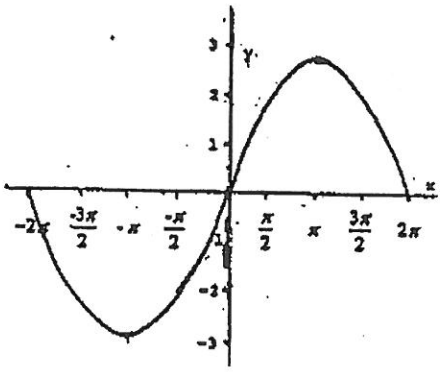


9. $\sec\theta = -\frac{\sqrt{13}}{3}$

10. $\theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

11. $\frac{27\pi}{45} = \frac{3\pi}{5}$

12. $-\frac{\sqrt{3}}{2}$

Topic VII - LOGARITHMIC AND EXPONENTIAL FUNCTIONS

1. $x = a^b$

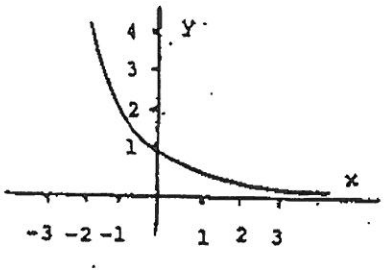
2. $u = \frac{\log 4}{\log 3} = 1.26$

3. $x = \frac{1}{4}$

4. $x = -\frac{23}{2}$

5. $5 - \log_2 60$

6. $\log(x^{3/2}y^4)$



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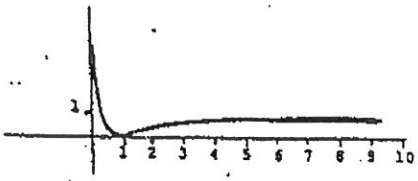

8.

9. $x = 0$

10. $x = 333$

Topic VIII - WORD PROBLEMS

1. $x = y + 2$
   $x - 15 = 2(y - 15)$

2. $l = 11\frac{2}{3}$

3. 25 times

4, $7\frac{1}{3}$, $9\frac{2}{3}$

5. 27

6. $\theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$

7. $150.00

8. 44%