FINITELY BIG NUMBERS

Name___________________

9^{9^9} or in order to avoid ambiguity $9^{(9^9)} = 9^{387420489}$. That is 9 to the 9th power of 9. Try plugin this number to your calculator and it cannot handle it. We can try and understand this process using small digits, such as 3.

$$3^{(3^3)} = 3^{27} = 7{,}625{,}597{,}484{,}987$$

But at the instant that we try to calculate $4^{(4^4)} = 4^{256}$ our calculators, even graphing calculators breaks down (even using a PC and MS excel 2010 cannot handle $5^{(5^5)} = 5^{3215}$ or beyond).

$9^{(9^9)} = 9^{387420489}$ has approximately 369, 692, 000 digits. In normal script, this number would be between 500 to 550 miles long. If you had a machine that would print one digit per second, printing the entire number would take approximately 11 years!

So even very shortly written finite numbers can be HUGE and a bit incomprehensible.

In order to put the big finite numbers in perspective here are some reference magnitudes that use the U.S. and modern British (short scale). Note: not all countries use the Short scale for numbering.

10^{6}	Million
10^{9}	Billion
10^{12}	Trillion
10^{15}	Quadrillion
10^{18}	Quintillion
10^{21}	Sextillion
10^{24}	Septillion
10^{27}	Octillion
10^{30}	Nonillion
10^{33}	Decillion

10^{63}	Vigintillion
10^{93}	Trigintillion
10^{123}	Quadragintillion
10^{153}	Quinquagintillion
10^{183}	Sexagintillion
10^{213}	Septuagintillion
10^{243}	Octogintillion
10^{273}	Nonagintillion
10^{303}	Centillion
10^{363}	Viginticentillion

10^{603}	Ducentillion
10^{903}	Trecentillion
10^{1203}	Quadringentillion
10^{1503}	Quingentillion
10^{1803}	Sescentillion
10^{2103}	Septingentillion
10^{2403}	Octingentillion
10^{2703}	Nongentillion
10^{3003}	Millinillion

http://en.wikipedia.org/wiki/Names_of_large_numbers
http://en.wikipedia.org/wiki/Orders_of_magnitude_(numbers)

Now let's put other numbers that are incomprehensibly big, but now you have a scale of (minor) comparison. The lower you go in the list the bigger the number gets.

Googol (Ten duotrigintillion) 10^{100} that is, the digit 1 followed by 100 zeroes:

10,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000.

Googolplex $10^{\text{googol}} = 10^{10^{100}}$

Skewes' number $10^{10^{10^{963}}}$.

Moser's number "2 in a mega-gon" is approximately equal to 10↑↑↑...↑↑↑10, where there are 10↑↑257 arrows. Therefore Moser's number, although incomprehensibly large, is vanishingly small compared to

Graham's number: $\text{moser} \ll 3 \rightarrow 3 \rightarrow 64 \rightarrow 2 < f^{64}(4) = \text{Graham's number.}$

TREE(3): appears in relation to a theorem on trees in graph theory. Representation of the number is difficult, but one weak lower bound is $A^{A(187196)}(1)$, where A(n) is a version of the Ackermann function. Graham's number, for example, is approximately $A^{64}(4)$ which is much smaller than the lower bound $A^{A(187196)}(1)$.

SSCG(3): Friedman's SSCG sequence begins SSCG(0) = 2, SSCG(1) = 5, but then grows rapidly. SSCG(2) = 3 × 23 × 295 − 9 ≈ 103.5775 × 1028. SSCG(3) is not only larger than TREE(3), it is much, much larger than TREE(TREE(…TREE(3)…)) where the total nesting depth of the formula is TREE(3) levels of the TREE function.

Rayo's number: The smallest number bigger than any number that can be named by an expression in the language of first order set-theory with less than a googol (10^{100}) symbols.

What follows is an introduction in Knuth's up-arrow notation, a method used in notation of very large numbers. http://en.wikipedia.org/wiki/Knuth's_up-arrow_notation . There are even other orders of notation that make up-arrow notation seem small as is in the case of Conway chained arrow notation http://en.wikipedia.org/wiki/Conway_chained_arrow_notation We will not into detail about the latter one.

Introduction The ordinary arithmetical operations of addition, multiplication and exponentiation are naturally extended into a sequence of hyperoperations as follows.

Multiplication by a natural number is defined as iterated addition:

$$a \times b = \underbrace{a + a + \cdots + a}_{b \text{ copies of } a}$$

For example,

$$4 \times 3 = \underbrace{4 + 4 + 4}_{3 \text{ copies of } 4} = 12$$

Exponentiation for a natural power b is defined as iterated multiplication, which Knuth denoted by a single up-arrow:

$$a \uparrow b = a^b = \underbrace{a \times a \times \cdots \times a}_{b \text{ copies of } a}$$

For example,

$$4 \uparrow 3 = 4^3 = \underbrace{4 \times 4 \times 4}_{3 \text{ copies of } 4} = 64$$

To extend the sequence of operations beyond exponentiation, Knuth defined a “double arrow” operator to denote iterated exponentiation (tetration):

$$a \uparrow\uparrow b = {}^{b}a = \underbrace{a^{a^{\cdot^{\cdot^{\cdot^{a}}}}}}_{b \text{ copies of } a} = \underbrace{a \uparrow (a \uparrow (\dots \uparrow a))}_{b \text{ copies of } a}$$

For example,

$$4 \uparrow\uparrow 3 = {}^{3}4 = \underbrace{4^{4^{4}}}_{3 \text{ copies of } 4} = \underbrace{4 \uparrow (4 \uparrow 4)}_{3 \text{ copies of } 4} = 4^{256} \approx 1.34078079 \times 10^{154}$$

Here and below evaluation is to take place from right to left, as Knuth's arrow operators (just like exponentiation) are defined to be right-associative.

According to this definition,

$$3 \uparrow\uparrow 2 = 3^3 = 27$$
$$3 \uparrow\uparrow 3 = 3^{3^3} = 3^{27} = 7,625,597,484,987$$
$$3 \uparrow\uparrow 4 = 3^{3^{3^3}} = 3^{7625597484987}$$
$$3 \uparrow\uparrow 5 = 3^{3^{3^{3^3}}} = 3^{3^{7625597484987}}$$

etc.

This already leads to some fairly large numbers, but Knuth extended the notation. He went on to define a “triple arrow” operator for iterated application of the “double arrow” operator (also known as pentation):

$$a \uparrow\uparrow\uparrow b = \underbrace{a \uparrow\uparrow (a \uparrow\uparrow (\dots \uparrow\uparrow a))}_{b \text{ copies of } a}$$

followed by a 'quadruple arrow' operator (also known as hexation):

$$a \uparrow\uparrow\uparrow\uparrow b = \underbrace{a \uparrow\uparrow\uparrow (a \uparrow\uparrow\uparrow (\dots \uparrow\uparrow\uparrow a))}_{b \text{ copies of } a}$$

and so on. The general rule is that an -arrow operator expands into a right-associative series of ()-arrow operators. Symbolically,

$$a \underbrace{\uparrow\uparrow\dots\uparrow}_{n} b = \underbrace{a \underbrace{\uparrow\dots\uparrow}_{n-1} (a \underbrace{\uparrow\dots\uparrow}_{n-1} (\dots \underbrace{\uparrow\dots\uparrow}_{n-1} a))}_{b \text{ copies of } a}$$

Examples:

$$3 \uparrow\uparrow\uparrow 2 = 3 \uparrow\uparrow 3 = 3^{3^3} = 3^{27} = 7,625,597,484,987$$

$$3 \uparrow\uparrow\uparrow 3 = 3 \uparrow\uparrow 3 \uparrow\uparrow 3 = 3 \uparrow\uparrow (3 \uparrow 3 \uparrow 3) = \underbrace{3 \uparrow 3 \uparrow \dots \uparrow 3}_{3 \uparrow 3 \uparrow 3 \text{ copies of } 3} = \underbrace{3 \uparrow 3 \uparrow \dots \uparrow 3}_{7{,}625{,}597{,}484{,}987 \text{ copies of } 3}$$

The notation $a \uparrow^n b$ is commonly used to denote $a \uparrow\uparrow \dots \uparrow b$ with n arrows.

Moral of the story: **None of these numbers even come close to compare how “big” the first size of infinity really is.**

What is infinity? $\aleph_0$

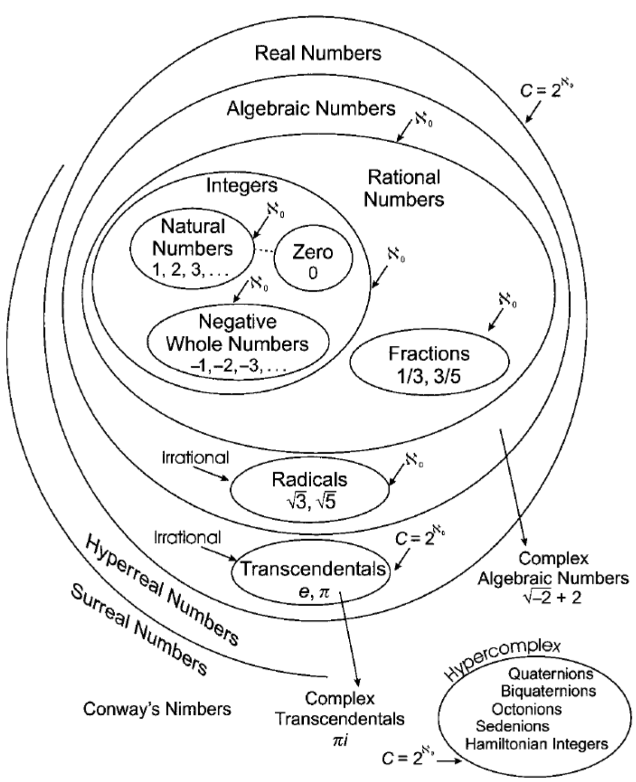
Infinity (symbol: ∞) refers to something without any limit. The term in the English language derives from Latin infinitas, which is translated as "unboundedness". In mathematics, "infinity" is often treated as if it were a number (i.e., it counts or measures things: "an infinite number of terms") but it is not the same sort of number as the real numbers. http://en.wikipedia.org/wiki/Infinity

In set theory, there are infinite sets of different sizes called cardinalities. The aleph numbers are a sequence of numbers used to represent the cardinality (or size) of infinite sets. They are named after the symbol used to denote them, the Hebrew letter aleph ($\aleph$).The cardinality of the natural numbers is $\aleph_0$ (read *aleph-naught*, *aleph-null* or *aleph-zero*), the next larger cardinality is aleph-one $\aleph_1$, then $\aleph_2$ and so on. Continuing in this manner, it is possible to define a cardinal number $\aleph_\alpha$ for every ordinal number α, as described below. http://en.wikipedia.org/wiki/Aleph_number

To read a very interesting story about how infinities can fit inside infinities of the same size you can read the “Hotel Infinity” story at the following site: http://www.c3.lanl.gov/mega-math/workbk/infinity/inhotel.html

And if you want to dive deeper into the rabbit hole you can see the Worlds Science Festival panel about infinity, where you can meet mathematicians that work daily with infinities.

https://www.youtube.com/watch?v=KDCJZ81PwVM



This is a great picture of the universe of numbers, but not complete. There are also p-adic numbers, Constructible number, periods, arithmetically definable, Dual numbers, hyperbolic numbers, Computable numbers, Supernatural numbers and others. http://en.wikipedia.org/wiki/Number

But it does gives you a visual representation on how the different number sets that we have studied interrelate among each other and their respective cardinalities.

Moral of the story: **There are an Infinite number of different Infinities, each “greater” than the previous infinity.**