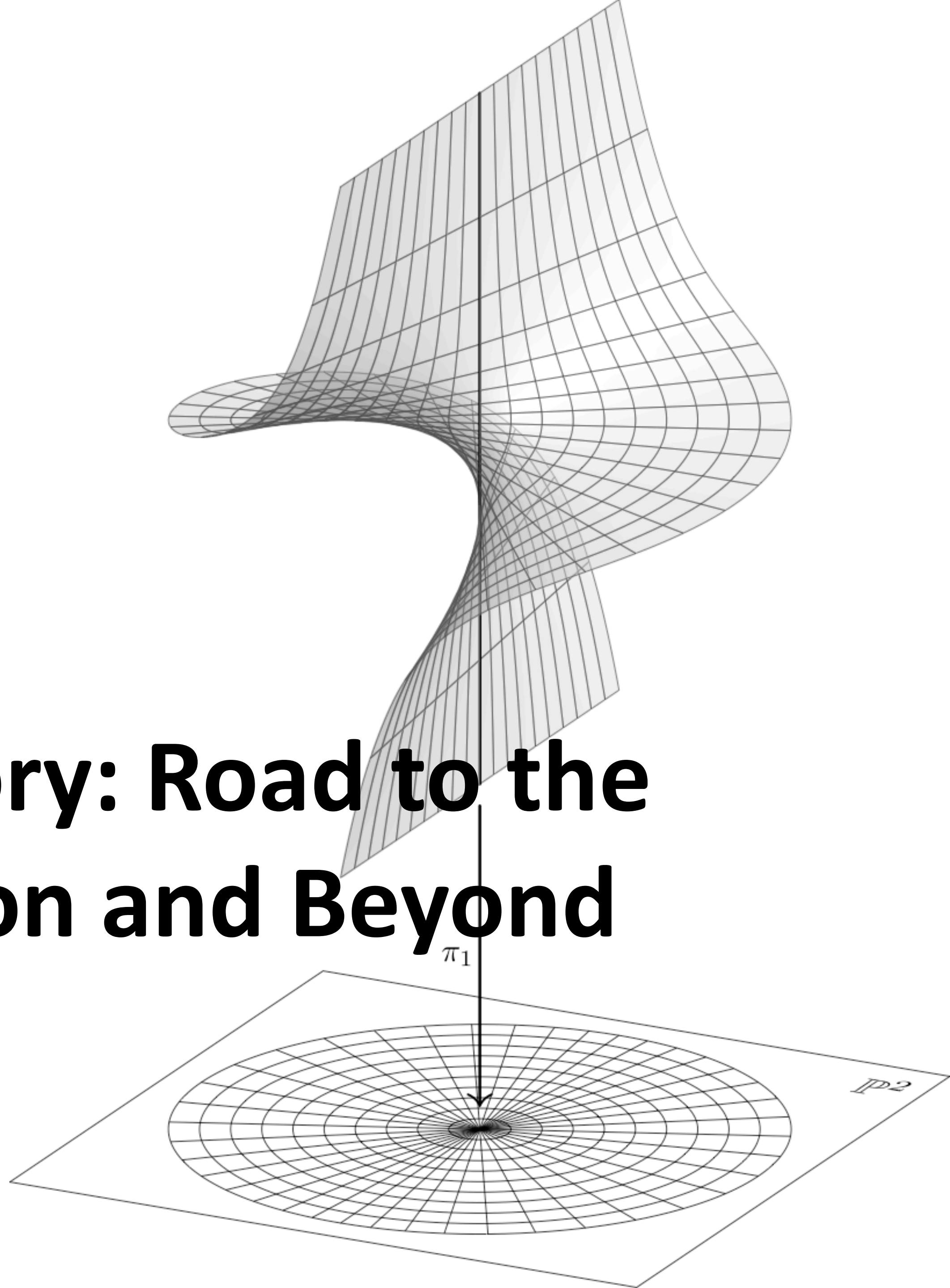
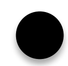
Dimension Theory: Road to the Fourth Dimension and Beyond



0-dimension

- "Behold yon miserable creature. That Point is a Being like ourselves, but confined to the non-dimensional Gulf. He is himself his own World, his own Universe; of any other than himself he can form no conception; he knows not Length, nor Breadth, nor Height, for he has had no experience of them; he has no cognizance even of the number Two; nor has he a thought of Plurality, for he is himself his One and All, being really Nothing. Yet mark his perfect self-contentment, and hence learn this lesson, that to be self-contented is to be vile and ignorant, and that to aspire is better than to be blindly and impotently happy."
- — Edwin A. Abbott, Flatland: A Romance of Many Dimensions

0-dimension



- Space of zero dimensions: A space that has no length, breadth or thickness (no length, height or width). There are zero degrees of freedom.
- The only “thing” is a point.
- $\{\ \} = \emptyset$

0-dimension

0-dimensional space.

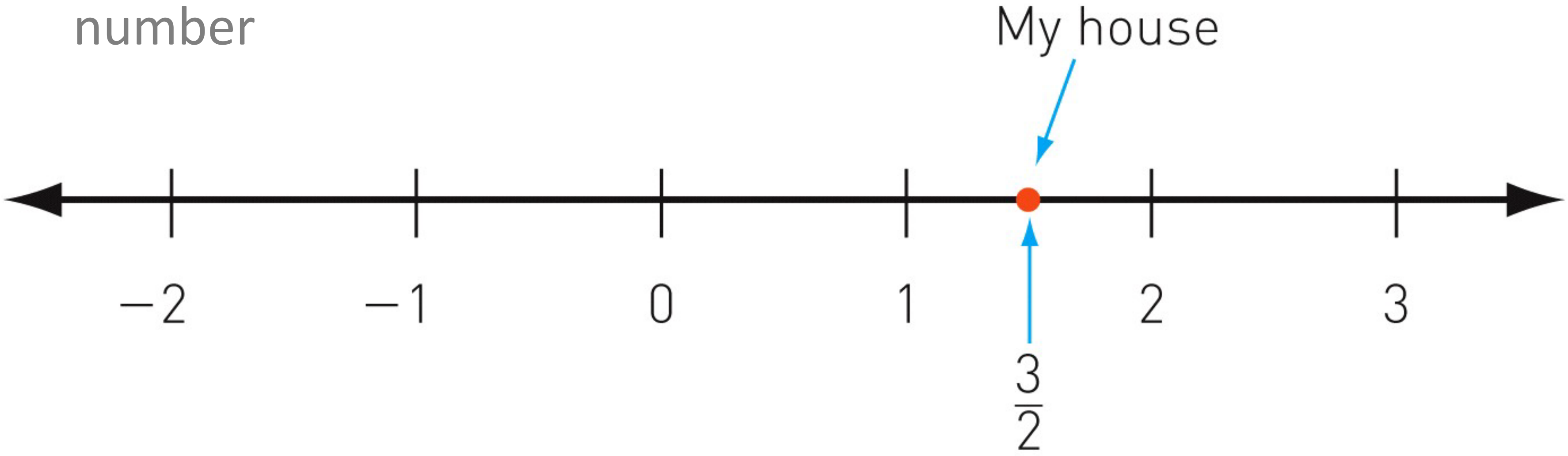
If you lived there,
you'd be home by now.

1-dimension

- Space of one dimension: A space that has length but no breadth or thickness
- A straight or curved line.
- Drag a multitude of zero dimensional points in new (perpendicular) direction
- Make a “line” of points

1-dimension

- One degree of freedom: Can only move right/left (forwards/backwards)
- $\{x\}$, any point on the number line can be described by one number



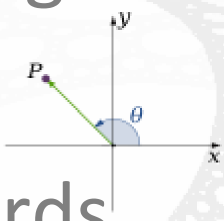
My address is $\frac{3}{2}$: one piece of information and you know where I live implies 1-dimensional space!

1-dimension

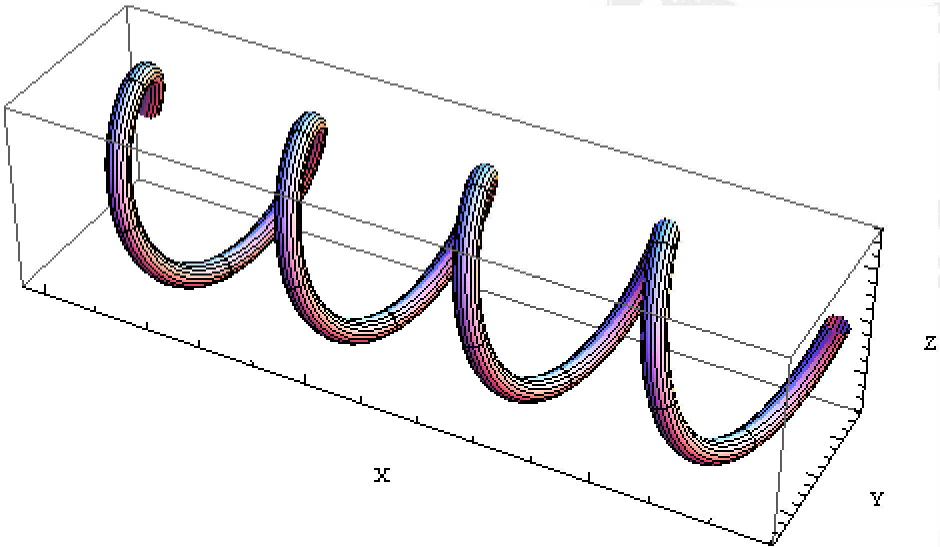
- How to visualize living in 1-dimension
- Stuck on an endless one-lane one-way road
- Inhabitants: points and line segments (intervals)
- Live forever between your front and back neighbor. Hope no one expels gas!
- “Morse code” from outside:

1-dimension

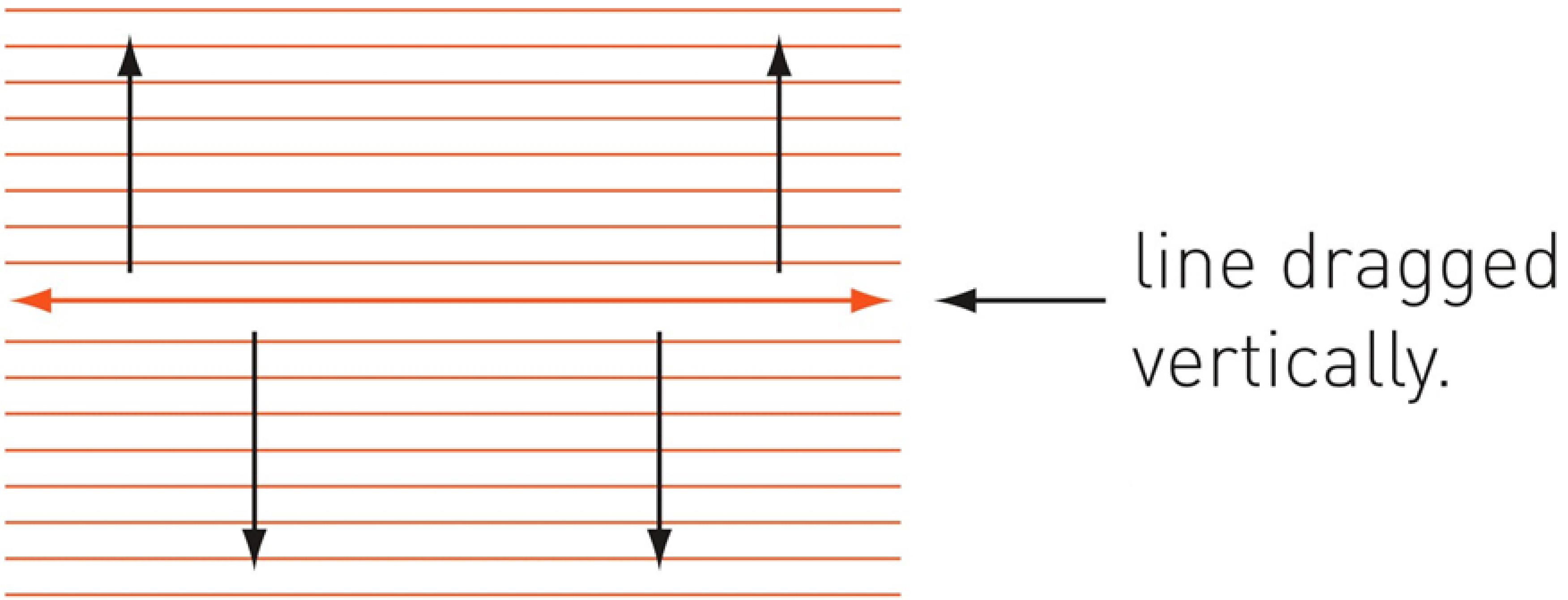
- Alternate interpretation
- Single number can be interpreted as an angle.
- $\{3.14159265\} = \{\pi\ radians\} = \{180°\}$



- Positive angle=counterclockwise or forwards
- Negative angle=clockwise or backwards



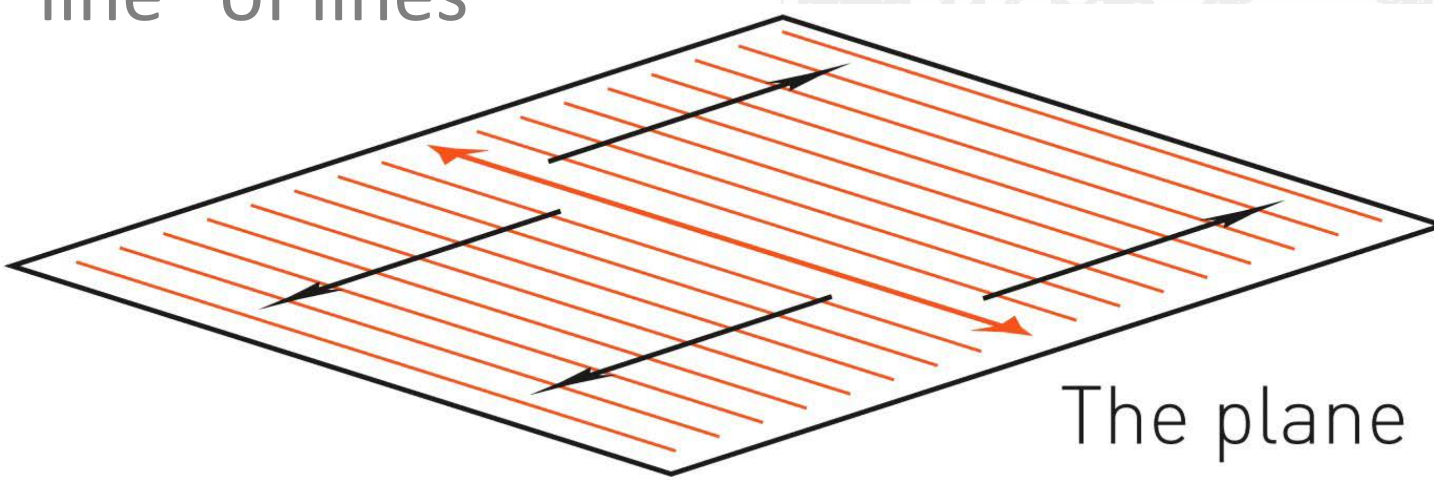
2-dimension



Produces the 2-dimensional plane.

2-dimension

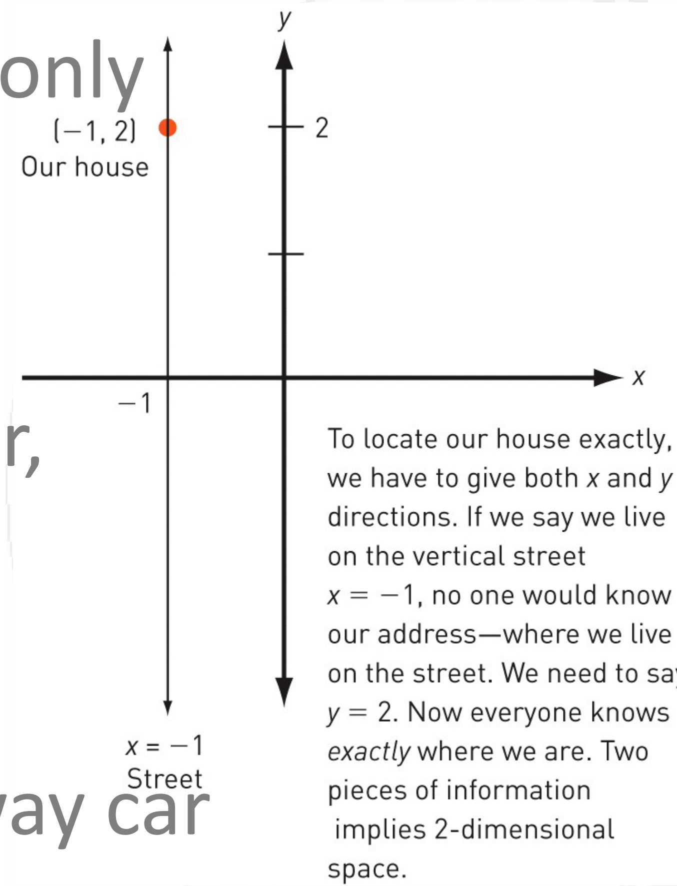
- Space of two dimensions: A space which has length and breadth, but no thickness; a plane or curved surface.
- Drag a multitude of one dimensional lines in new (perpendicular) direction
- Make a “line” of lines



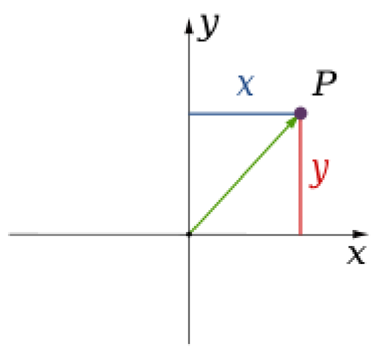
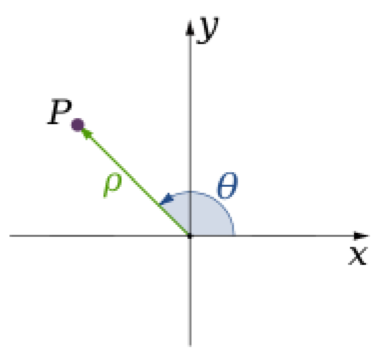
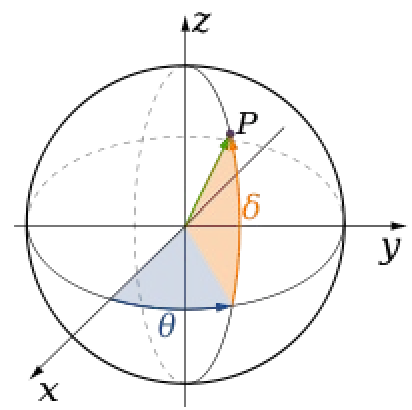
2-dimension

- (x, y), any point on the plane can be described by an order pair of numbers
- Two degrees of freedom: can only move right/left or up/down
- Boat analogy. Cannot escape

Surface of water, but have more freedom than one-lane one-way car



2-dimension

		
Cartesian (two-dimensional)	Polar (trigonometry)	Latitude and longitude (navigation)

- Alternate interpretations
- Polar Coordinates $(r, \theta) = (radius, angle)$
- GPS Coordinates $33°8'59.9676''N, 117°10'58.4472''W$

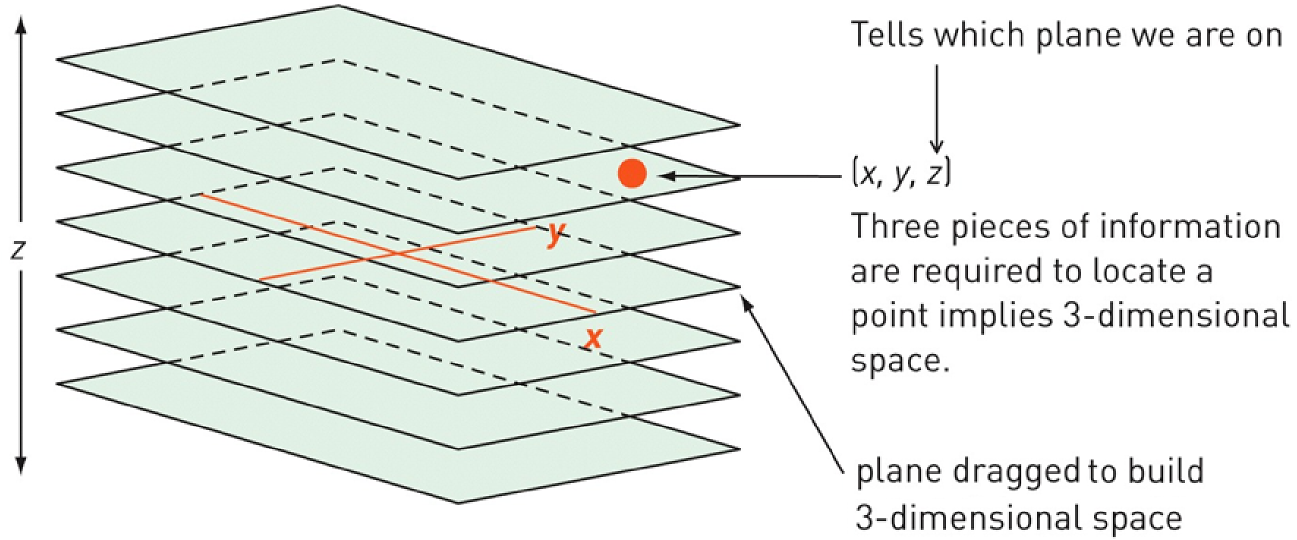
3-dimension

- “If I have seen further than others, it is by standing upon the shoulders of giants.”
- Isaac Newton

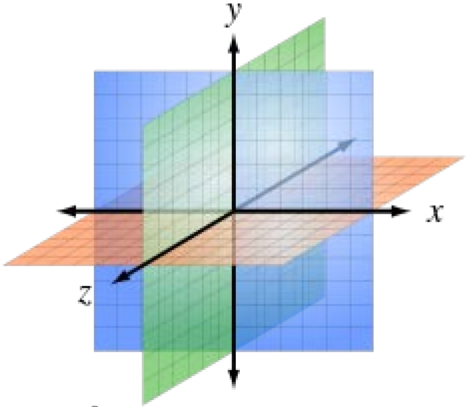
- *Reptiles 1943*
- (M. C. Escher)

3-dimension

- Space of three dimensions: A space which has length, breadth, and thickness; a solid.
- Drag a multitude of two dimensional planes in new (perpendicular) direction
- Make a “line” of planes

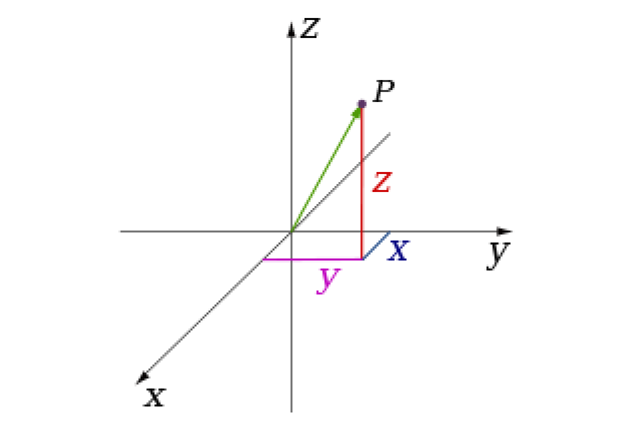


3-dimension

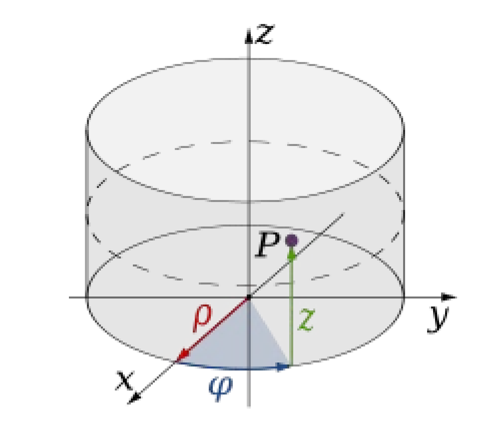


- (x,y,z), any point in 3D space can be described by an ordered triplet of numbers
- Three degrees of freedom: can only move , right/left, up/down and forwards/backwards
- Plane analogy: have more freedom than one-lane one-way car or boat on surface of ocean

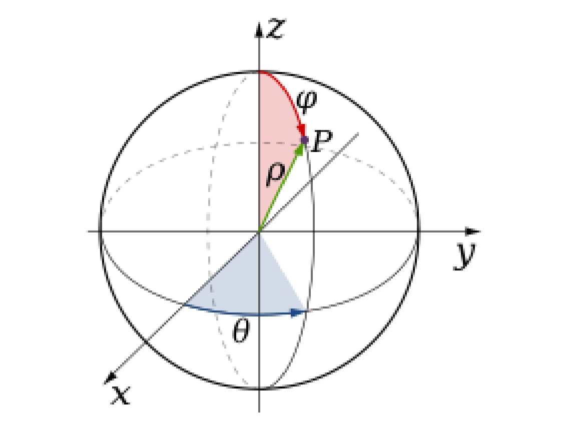
3-dimension



Cartesian (three-dimensional)



Cylindrical



Spherical

- Alternate interpretations
- Cylindrical coordinates (ρ, φ, z)=radial distance, azimuth angle and height
- Spherical (r, θ, φ)=radial distance, polar angle, azimuth angle

Visualizing higher dimensions from lower ones

- How can 2 dimensional beings understand a visit from a third dimensional "being"

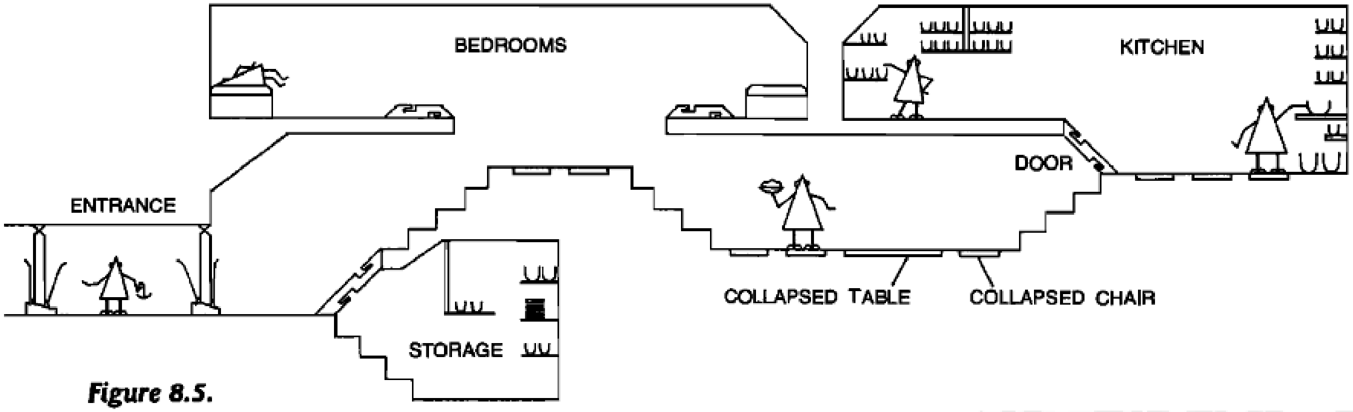
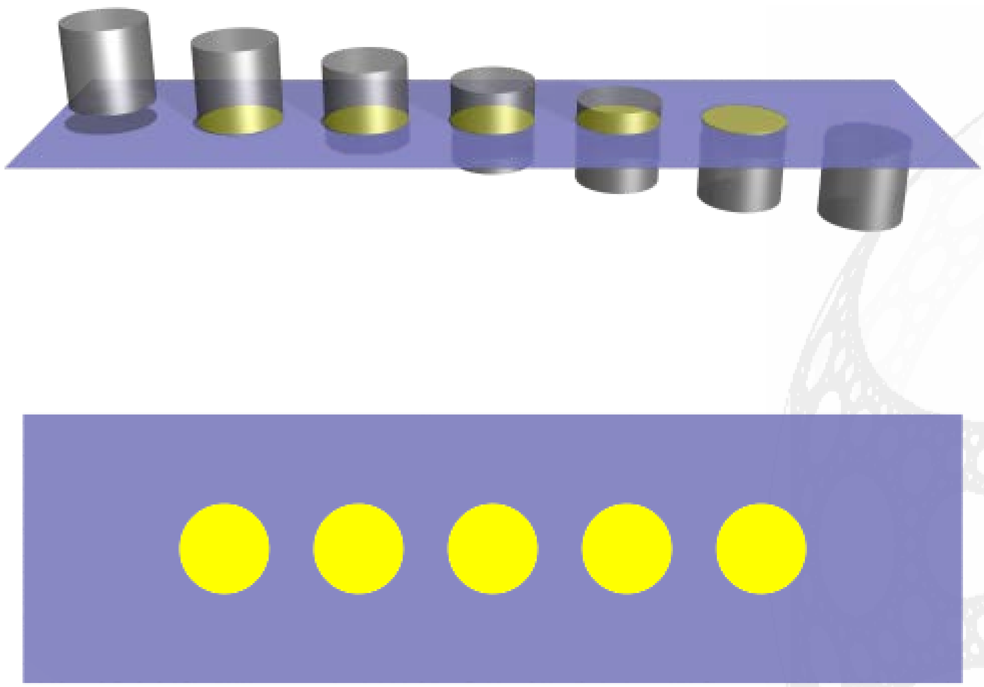


Figure 8.5.

- What will regular 3D solids look in their dimension?

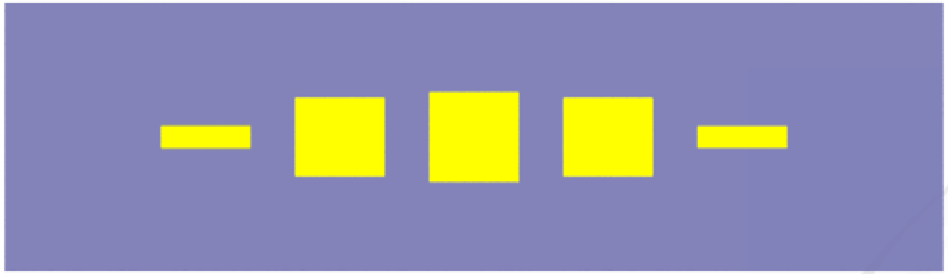
Visualizing higher dimensions from lower ones

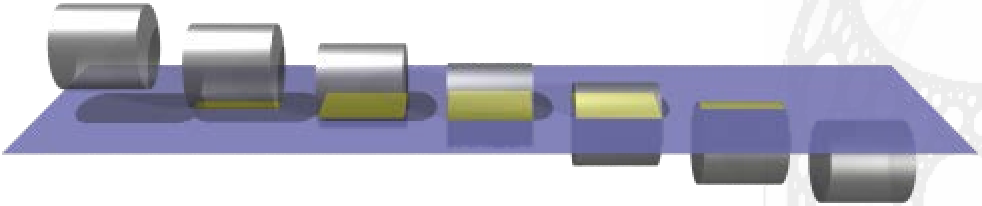
- What will a cylinder look like to a 2D being?



Visualizing higher dimensions from lower ones

- What is this object?

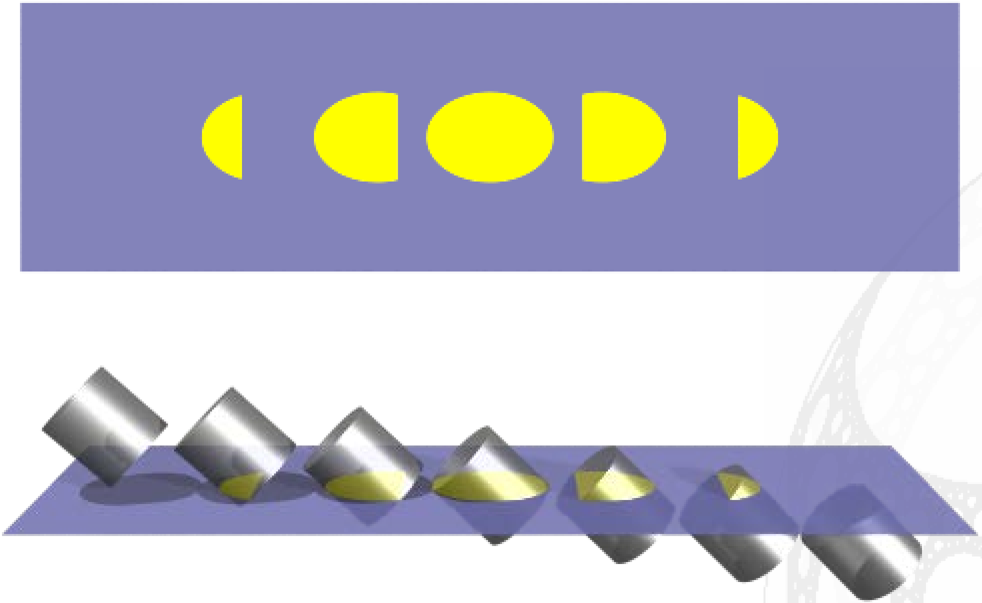




- Still a cylinder!!!
- We have to be careful to judge a "book by its cover"
- Not easy, but still possible to decipher object by its cross sections

Visualizing higher dimensions from lower ones

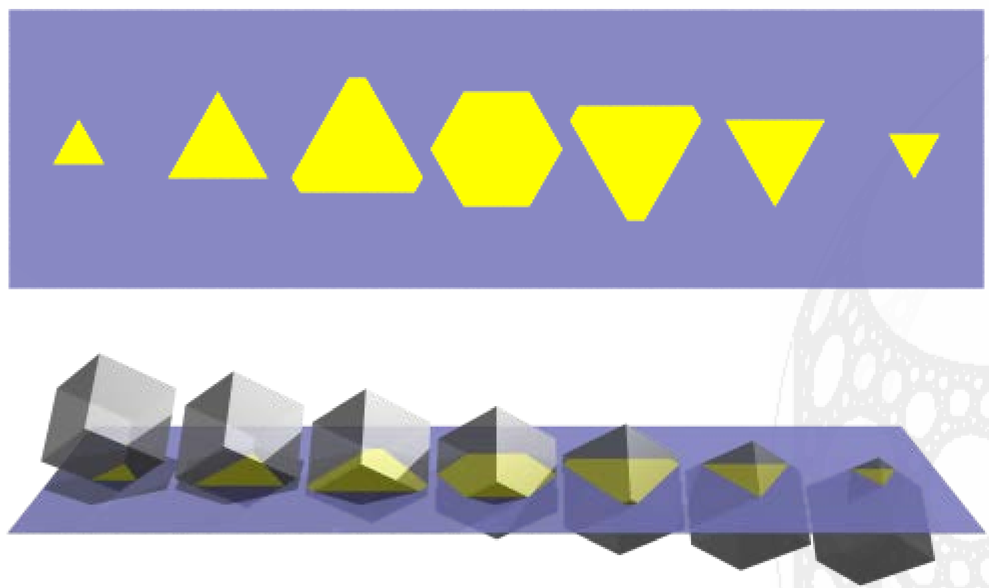
- What is this object?



- Still a cylinder!!!

Visualizing higher dimensions from lower ones

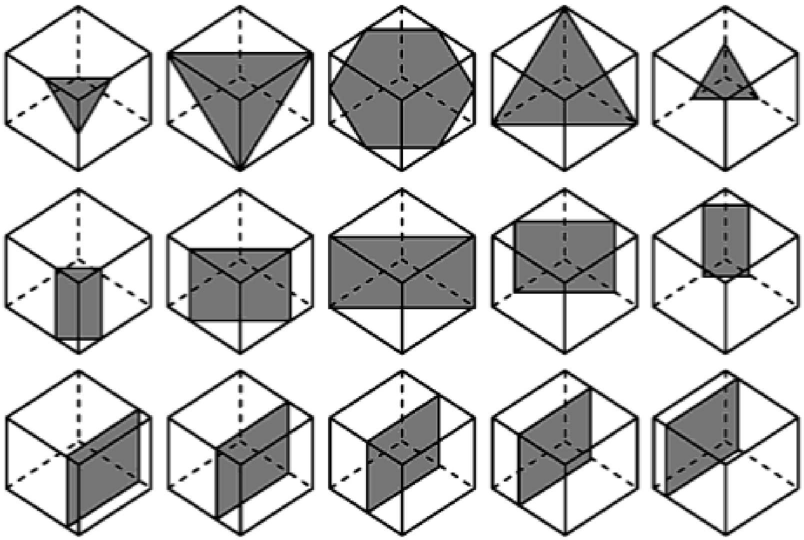
- Challenge: What is this object?
- Hint: not a cylinder anymore



- A cube!!!
- https://www.geogebra.org/m/M5dZnUeH#material/HSgSE469

Visualizing higher dimensions from lower ones

- More than one possible cross section "movie" sequence for the cube or any other solid

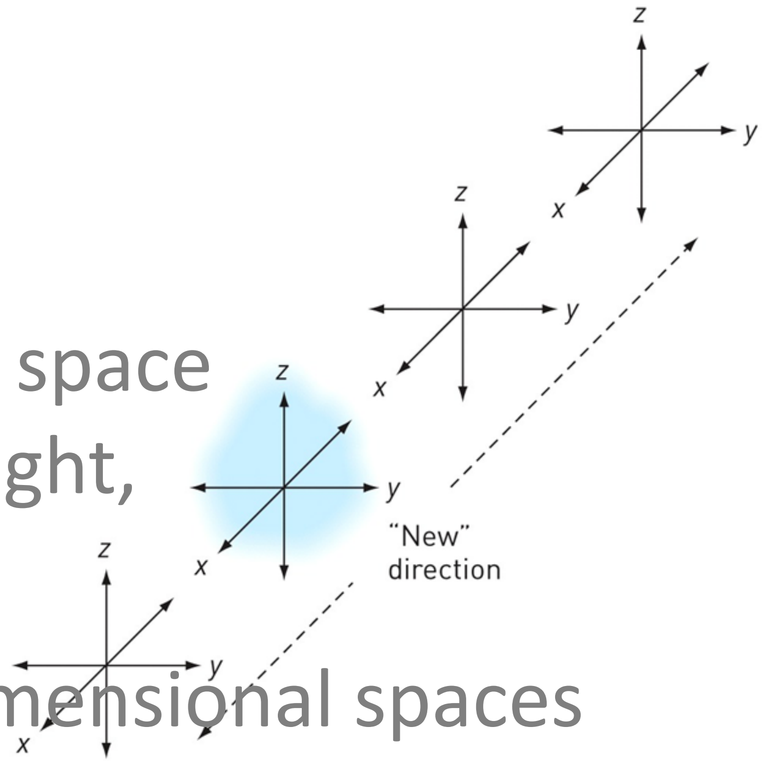


- Lets keep it simple and do "standard" projection analysis
- https://www.youtube.com/watch?v=AhM9JH5GNiI&index=2&list=PL3C690048E1531DC7

4-dimension

Max Weber, Interior of the Fourth Dimension, 1913.
National Gallery of Art

4-dimension



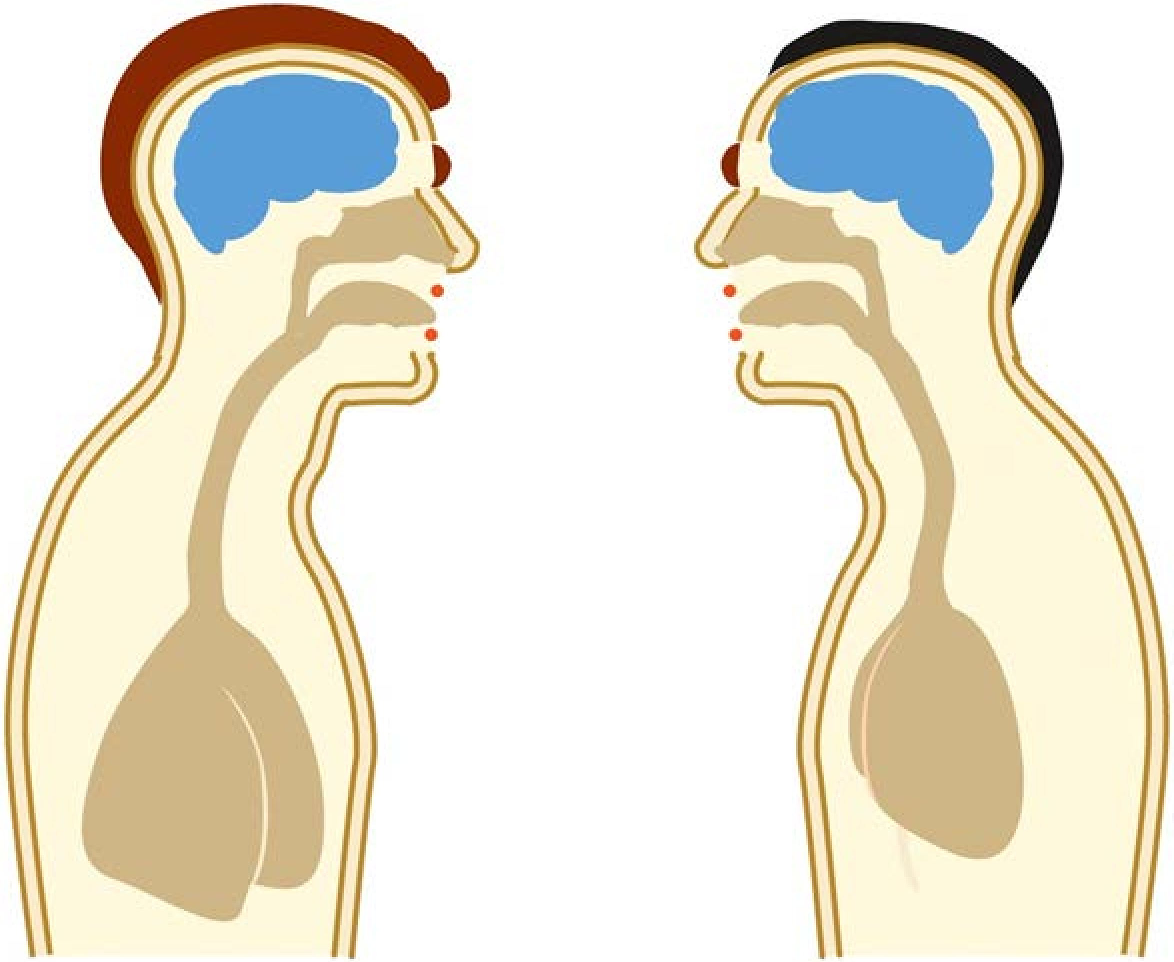
- Space of four dimensions: A space which has length, width, height, and trength; a hyper-solid.
- Drag a multitude of three dimensional spaces in new (perpendicular) direction
- Make a “line” of 3D spaces
- (x,y,z,w), any point in 4D sp
  described by an ordered qu
- Four degrees of freedom: ca
  up/down, forwards/backwards and **ana/kata**

dimension	measure	small measure	great measure
1st	length	short	long
2nd	width	narrow	wide
3rd	height	short	tall
4th	trength	tarrow	trong

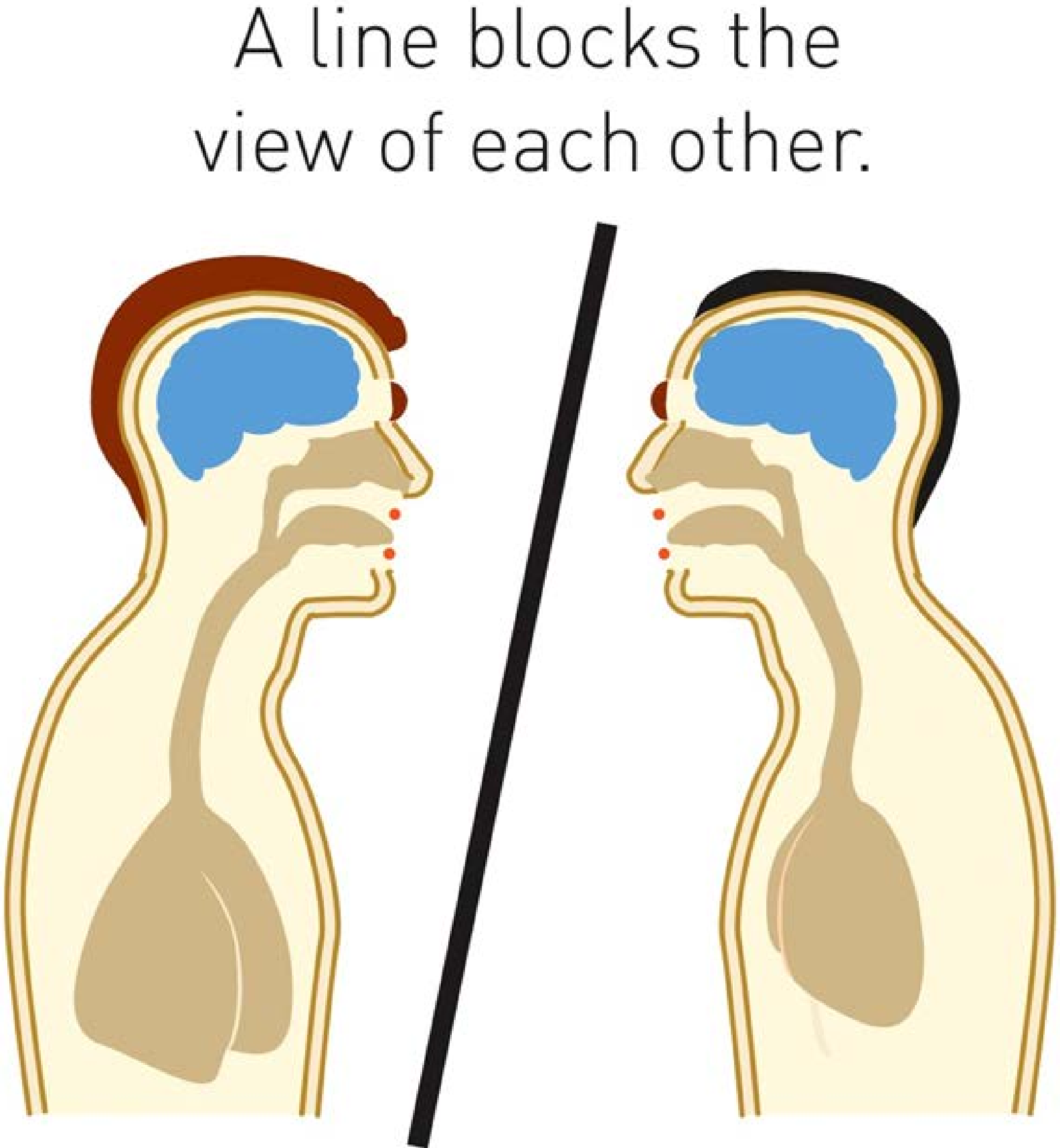
4-dimension

- https://www.youtube.com/watch?v=nz0ku71x22A&index=3&list=PL3C690048E1531DC7

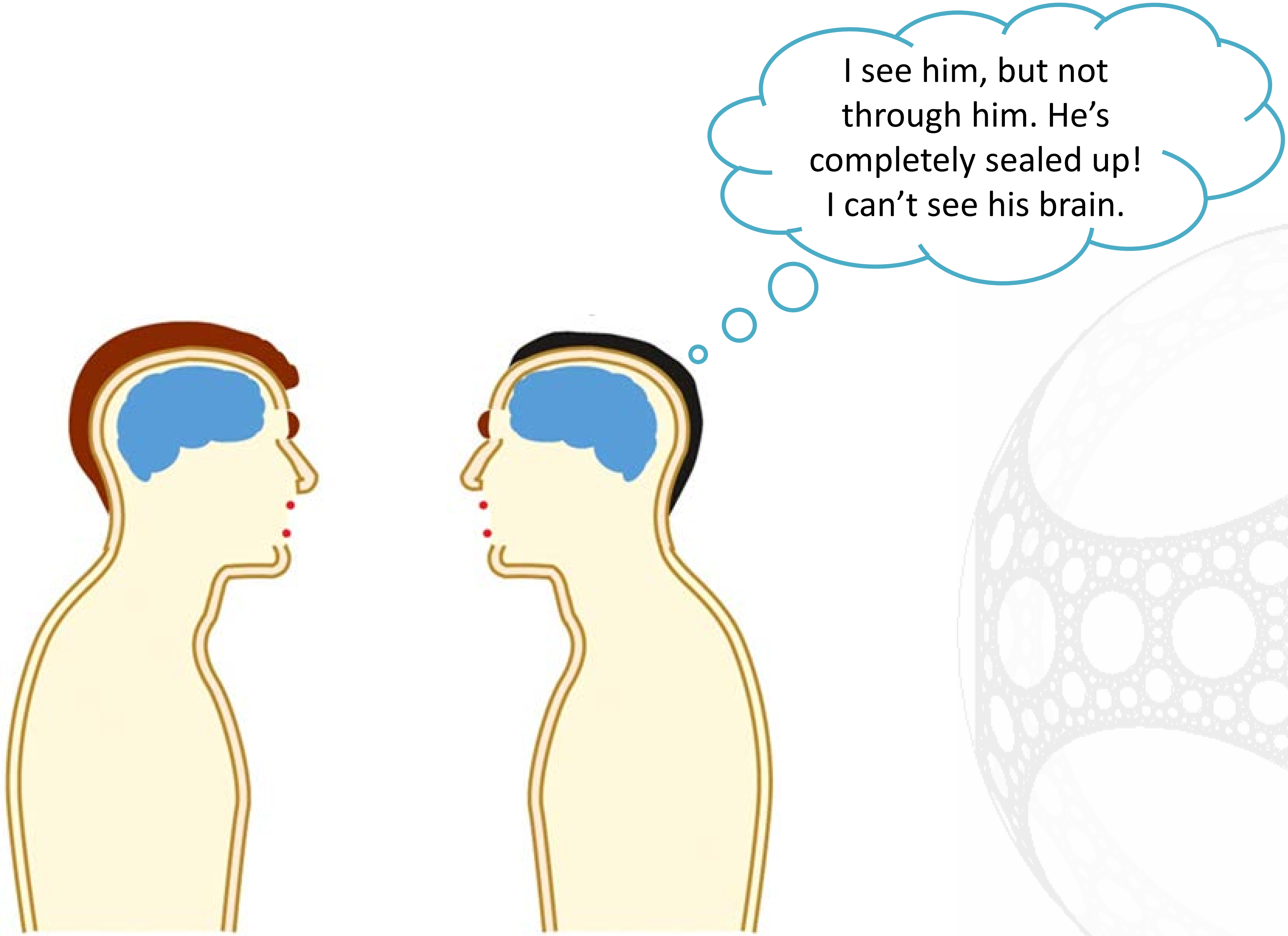
Mischief in lower dimensions



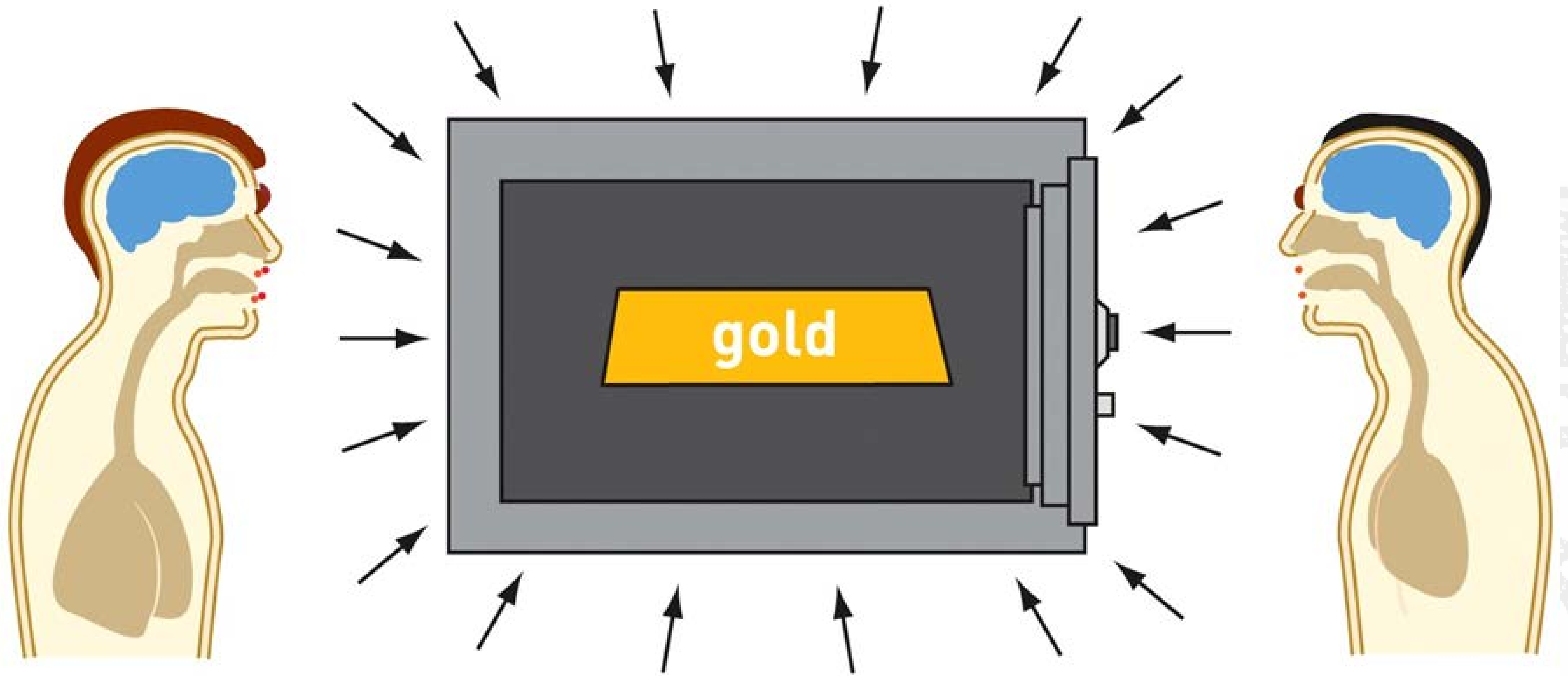
Mischief in lower dimensions



Mischief in lower dimensions

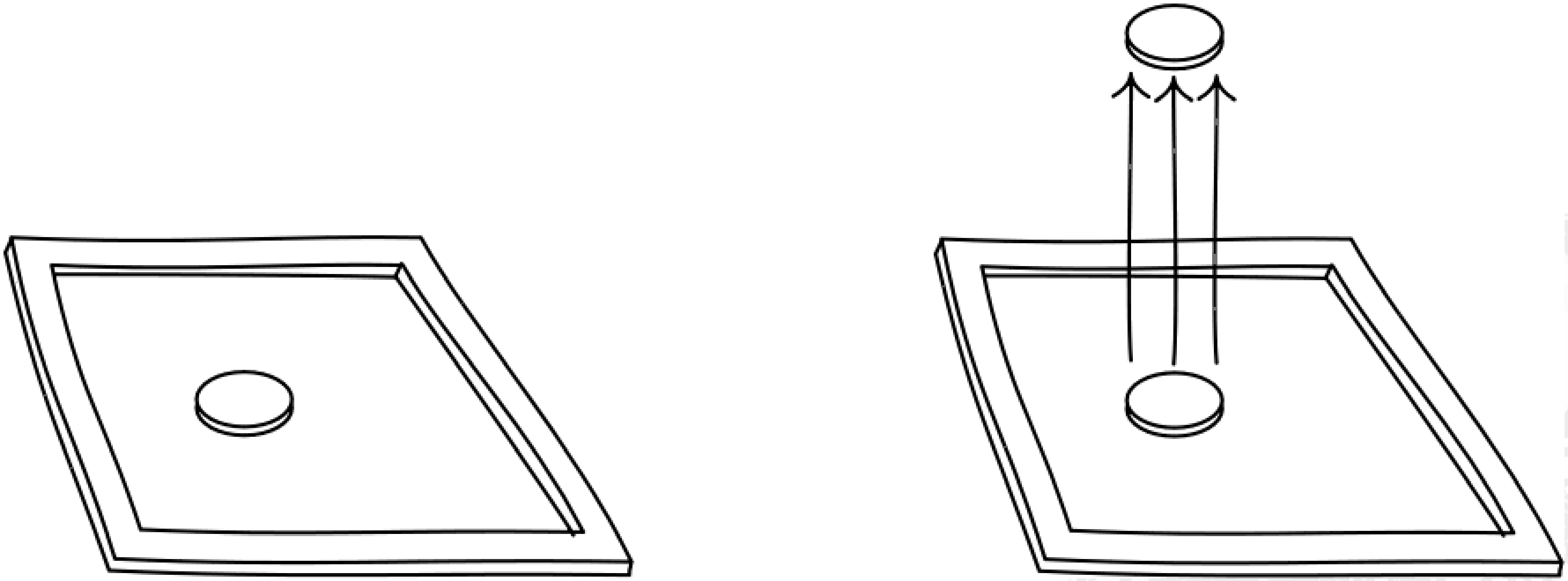


Mischief in lower dimensions



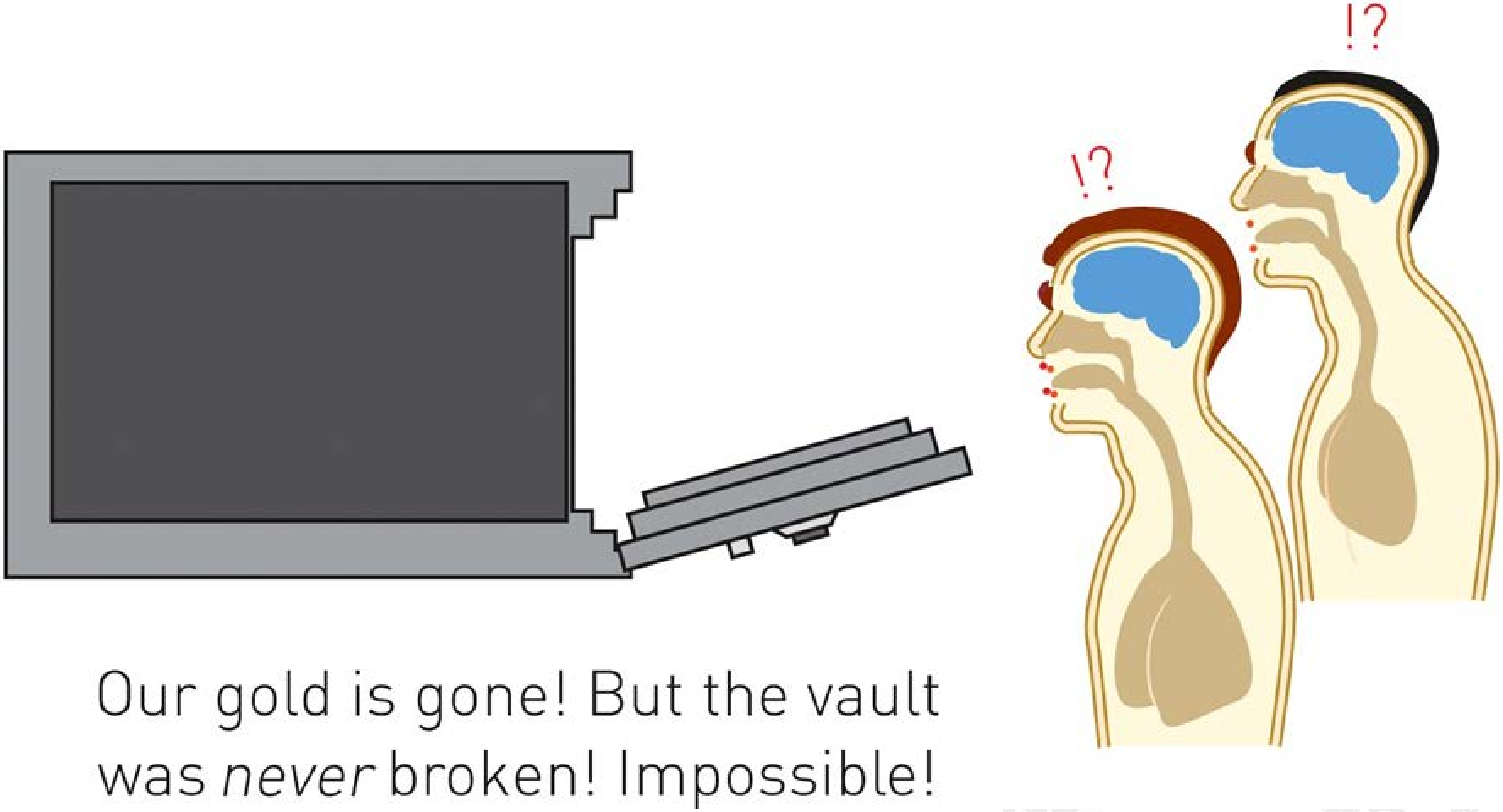
Our 2-dimensional vault — sealed ALL around!

Mischief in lower dimensions

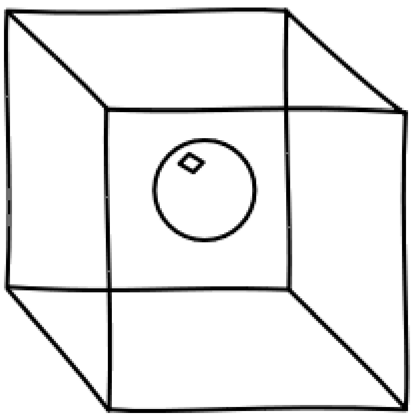


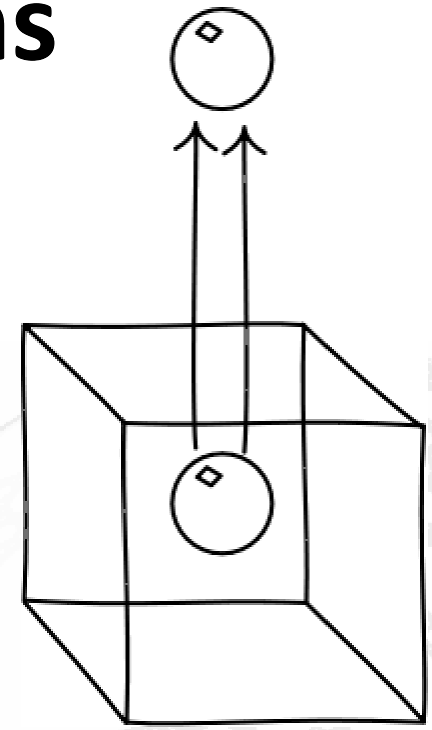
- Now way to remove the coin within the confines of the two dimensional surface of the table.
- There is no place to “lift it” If we only consider the 2 dimensions of the tables surface.

Mischief in lower dimensions



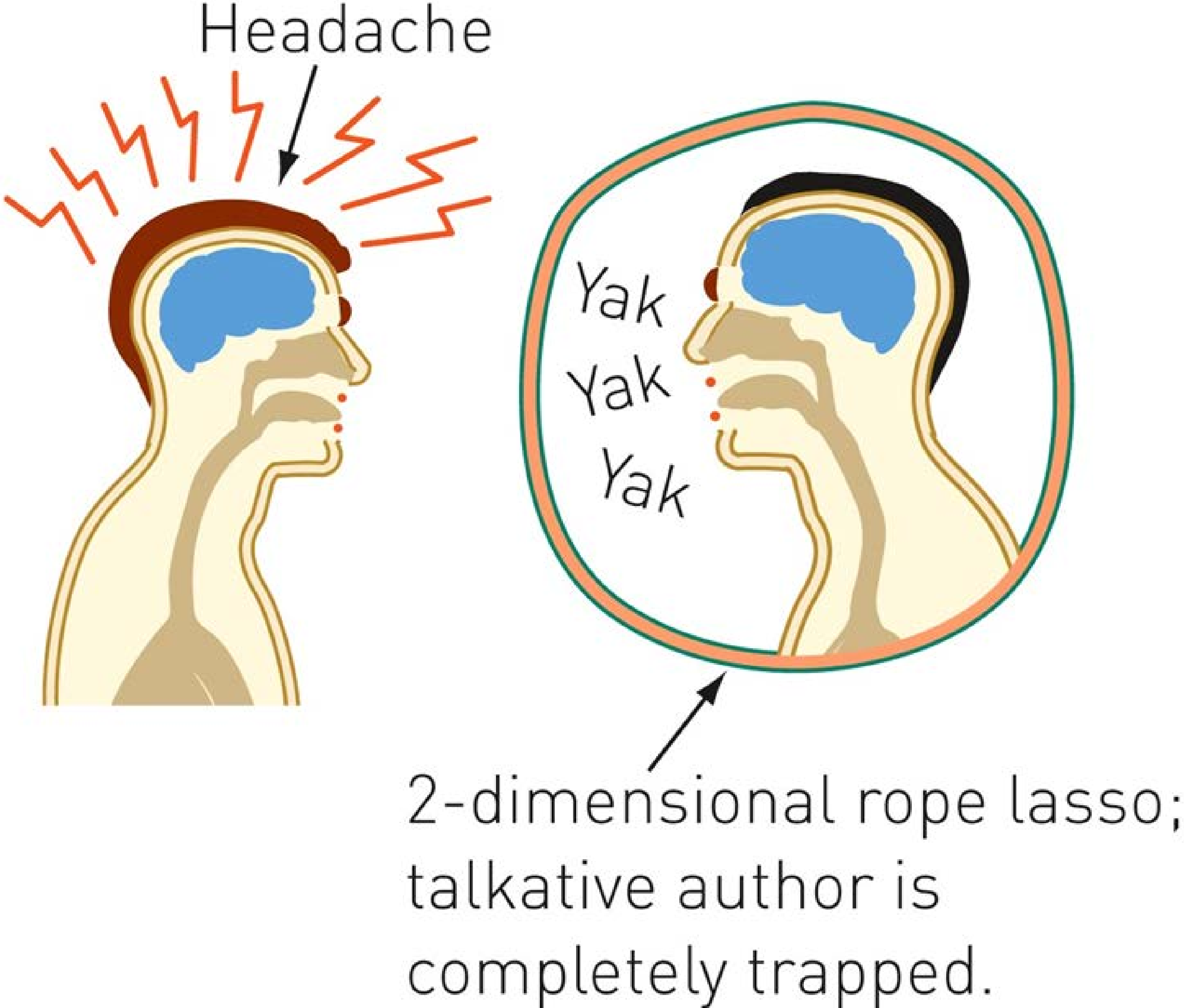
Mischief in lower dimensions



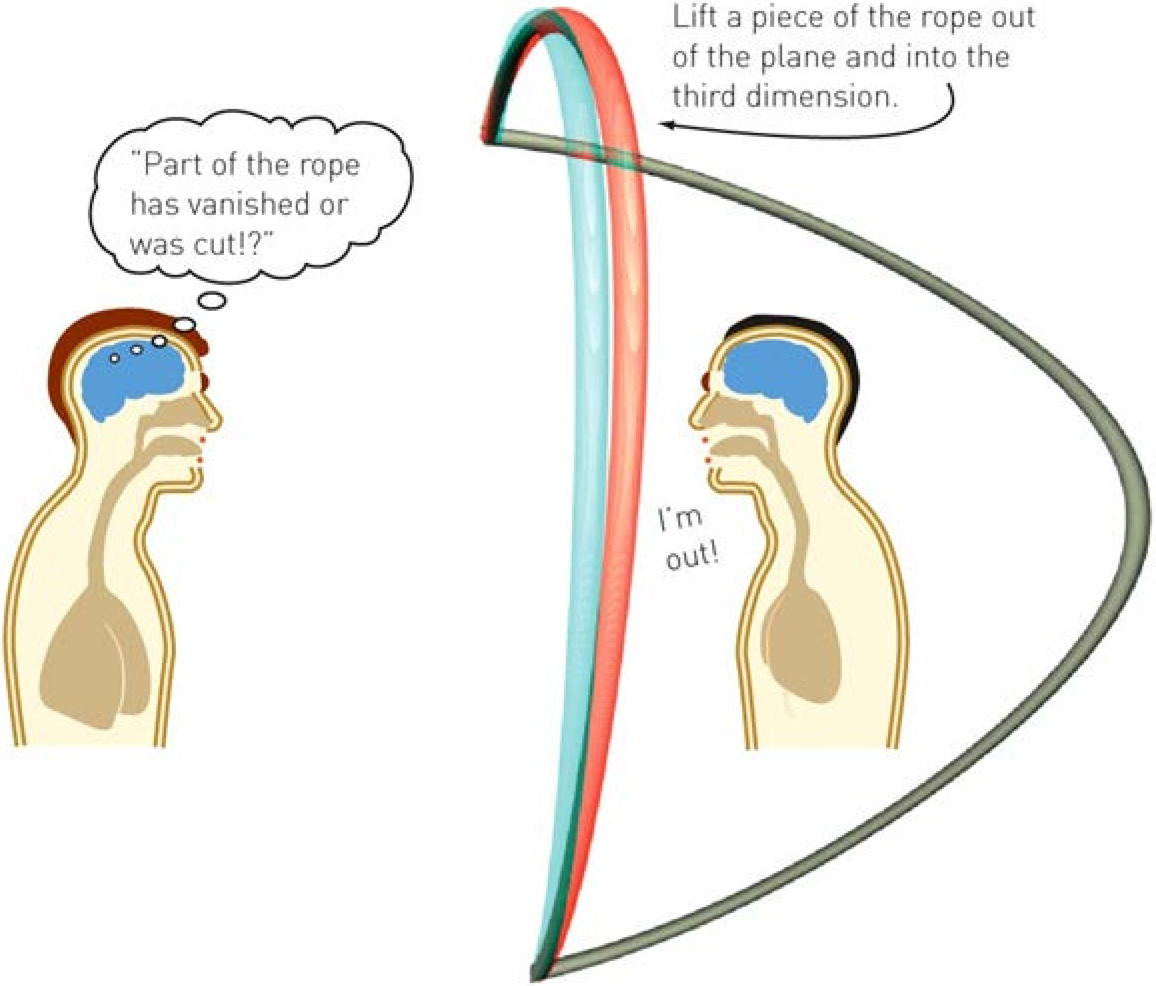


- Now, a marble trapped within a three dimensional sealed box. (or soda bottle inside a sealed refrigerator, or all the gold locked inside The U.S. Gold Bullion Depository, aka fort Knox.)

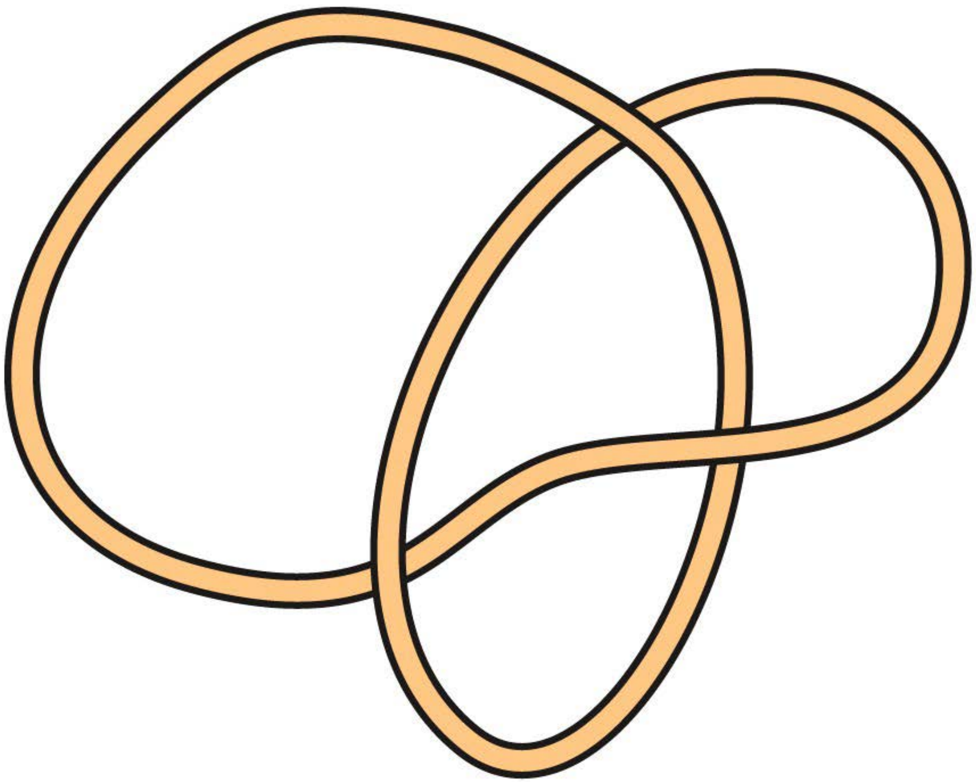
Mischief in lower dimensions



Mischief in lower dimensions

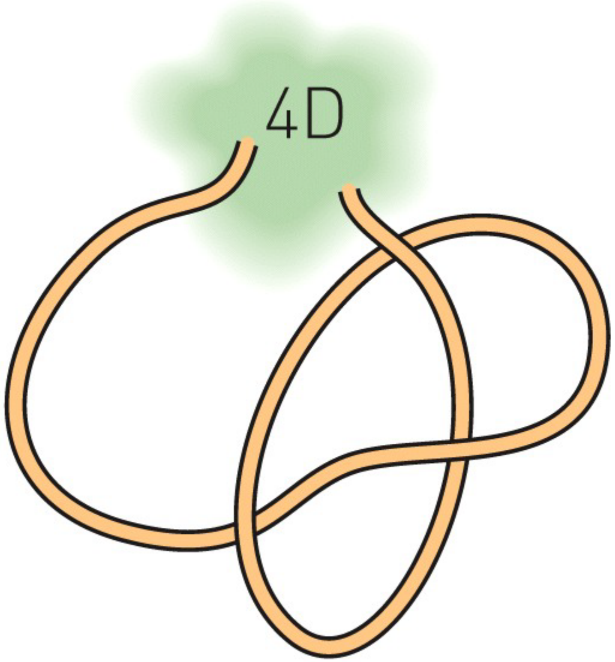


Mischief in lower dimensions



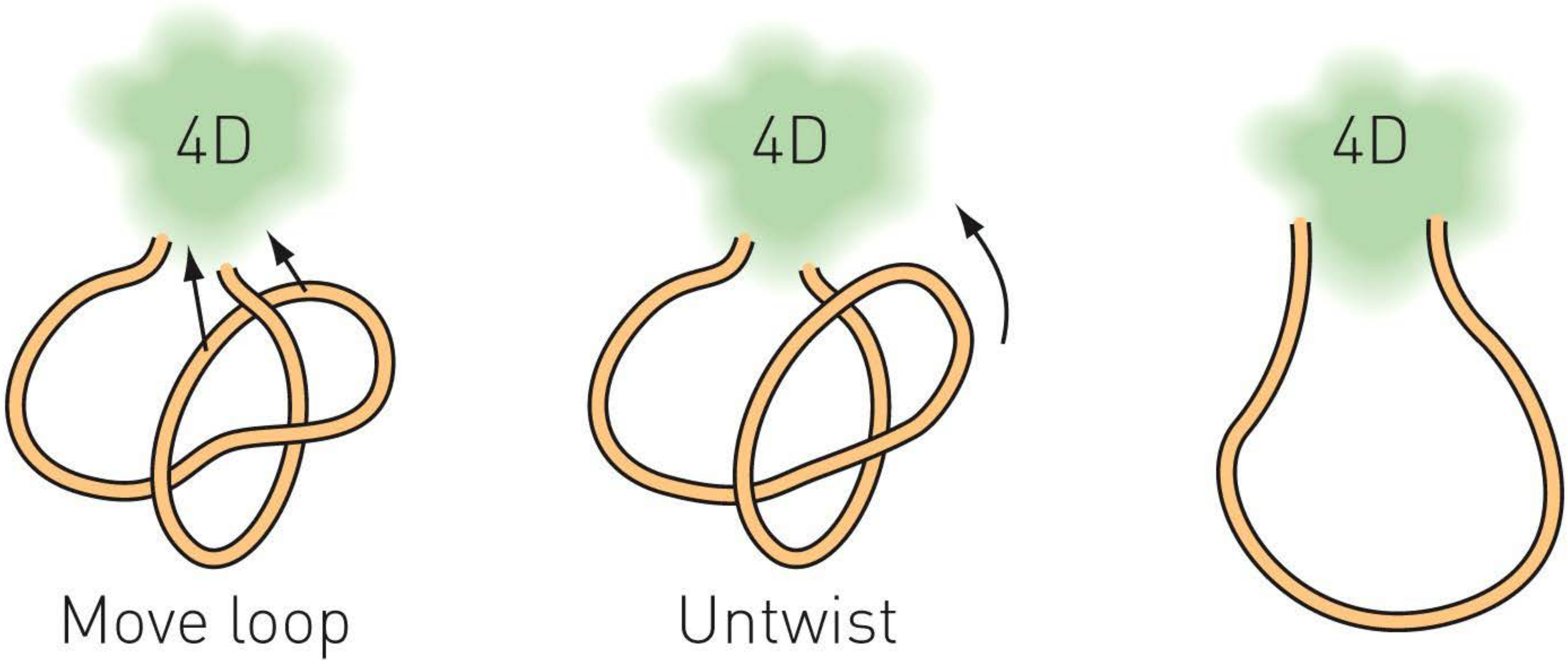
Knotted rope loop
in three dimensions

Mischief in lower dimensions



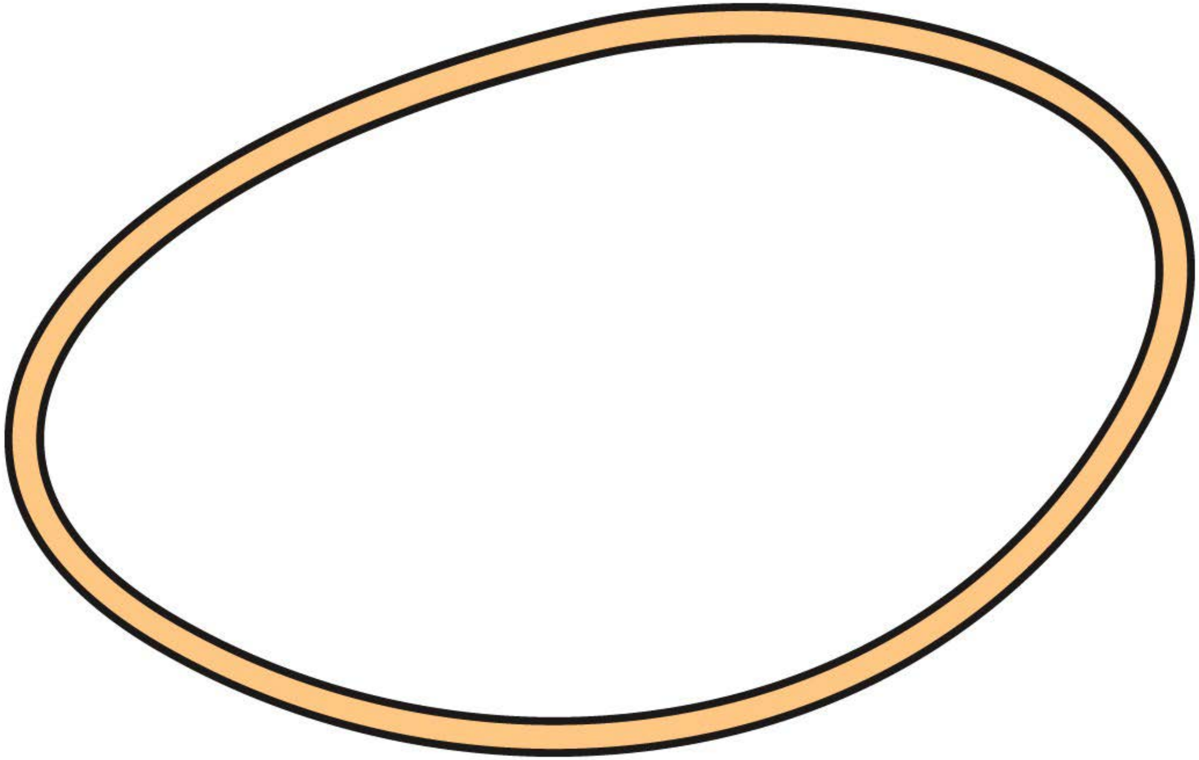
Fredddd lifts a piece of the rope up into the fourth dimension. Is the rope cut? No! But from our 3-dimensional vantage point, it's open. So, . . .

Mischief in lower dimensions



Now Fredddd lowers back the piece of rope he was holding in the fourth dimension ⟶ we see the rope "fuse" together magically.

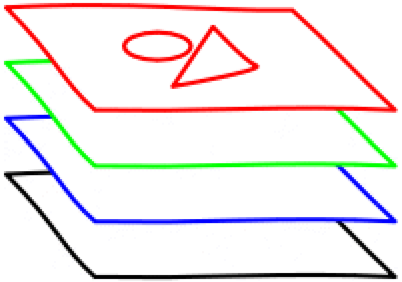
Mischief in lower dimensions



Unknotted rope!

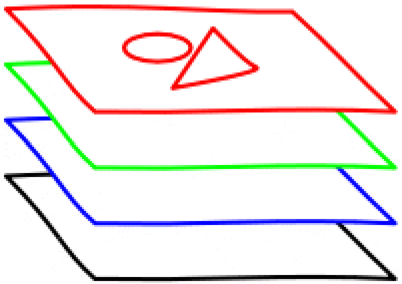
No cutting.

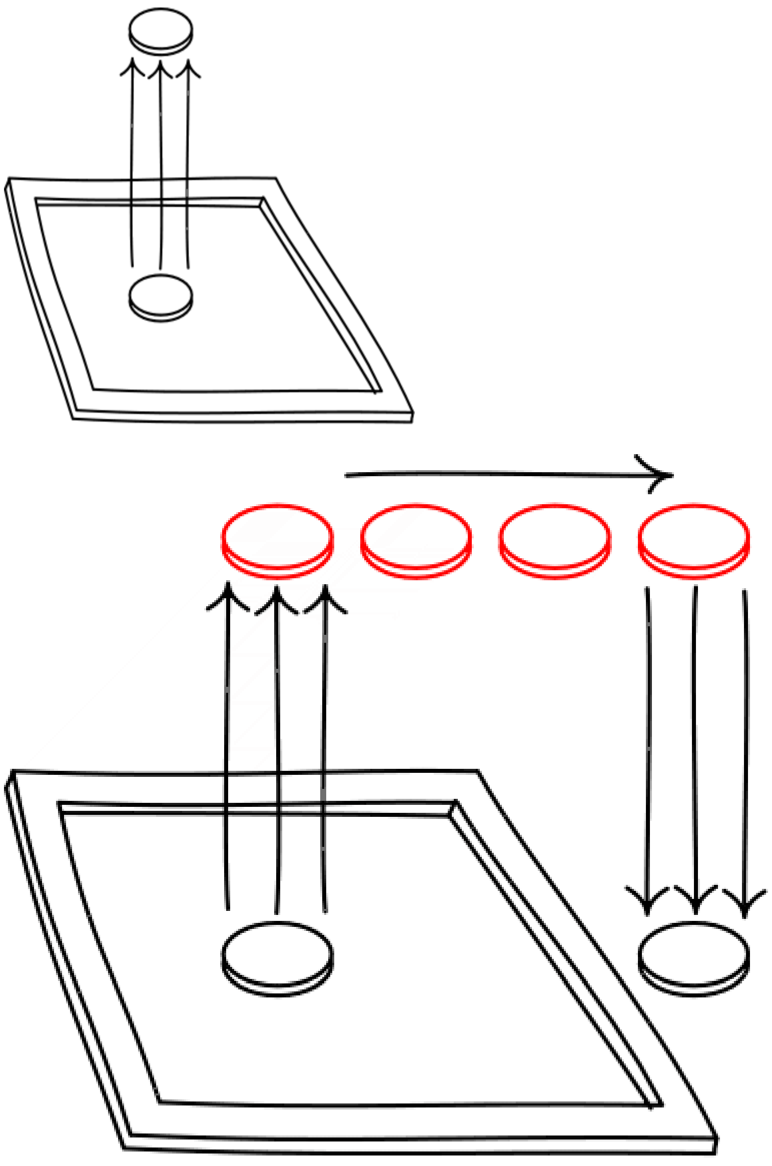
4^{th}-dimension as color

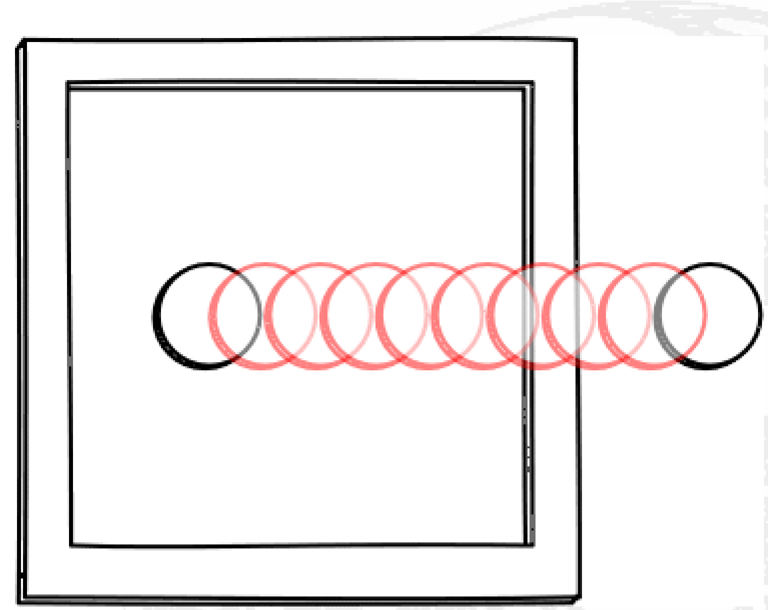


- Imagine that *differences in position* in the extra dimension of space can be represented by differences of colors.
- Visualizing lifting into the third dimension
- The objects in the original two dimensional space are black.
- As we lift through the third dimension, they successively take on the colors blue, green and red.

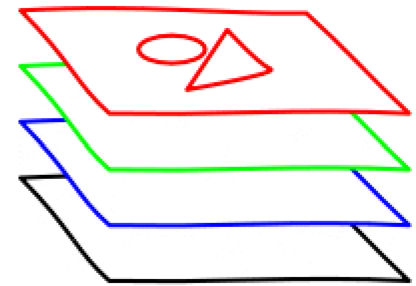
4th-dimension as color

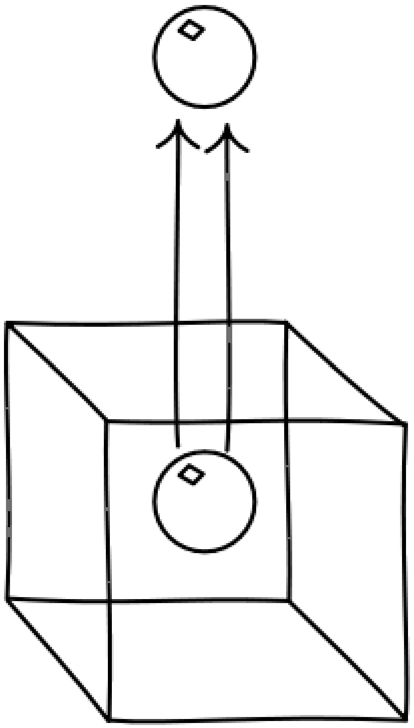


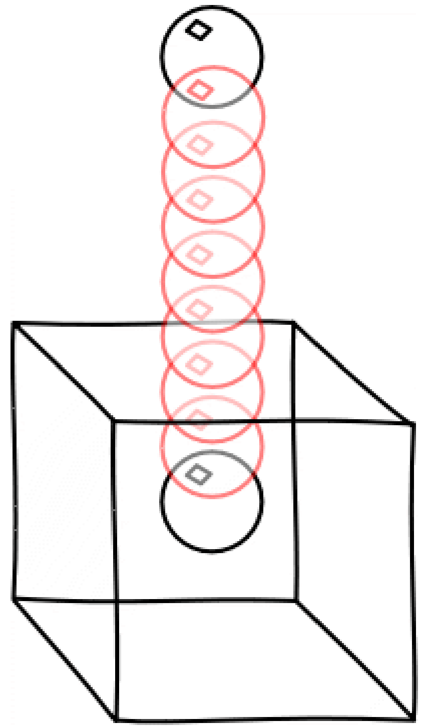




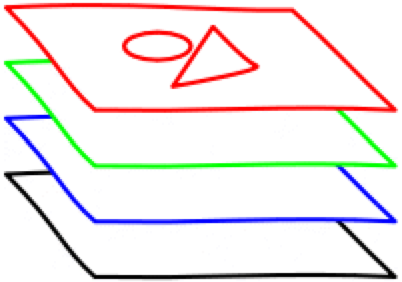
4^{th}-dimension as color



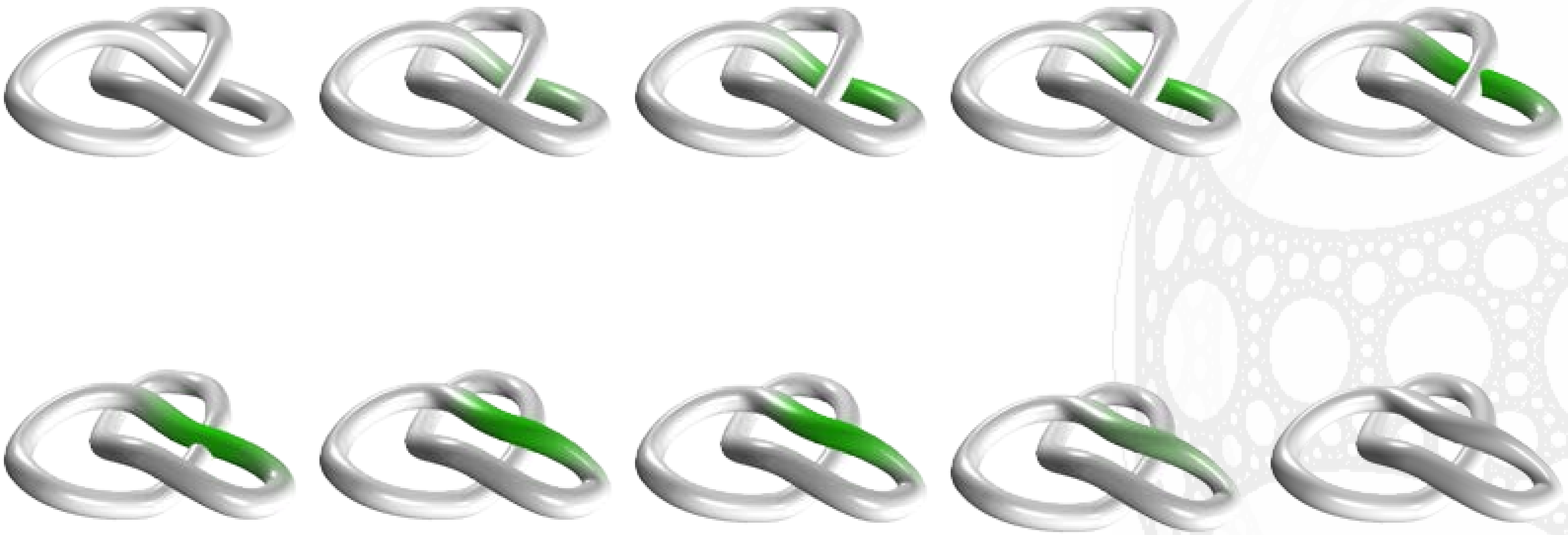




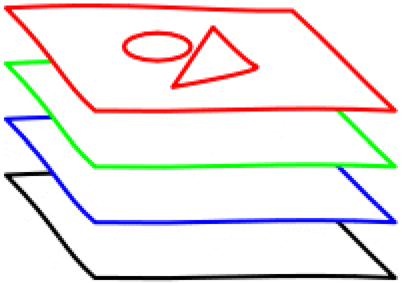
4th-dimension as color



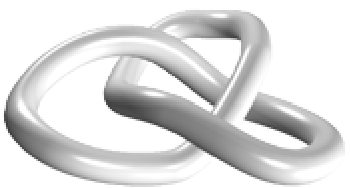
- How to unknot a rope using the 4th dimension



4th-dimension as color

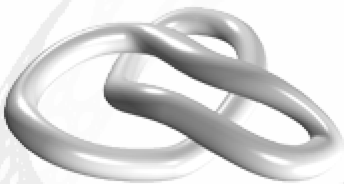


- If we witness this knot untying before our eyes, what do will we actually experience?



Starts completely in 3D space

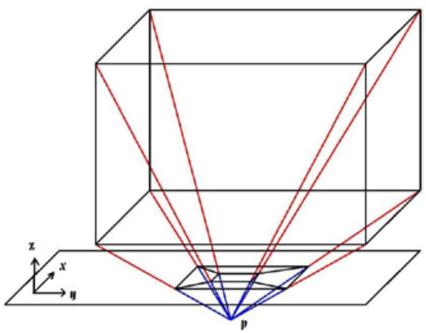
Loop moving through 4D space
-- not visible to us

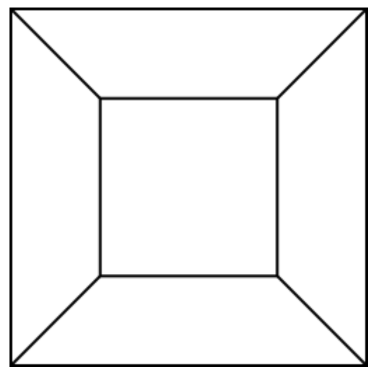


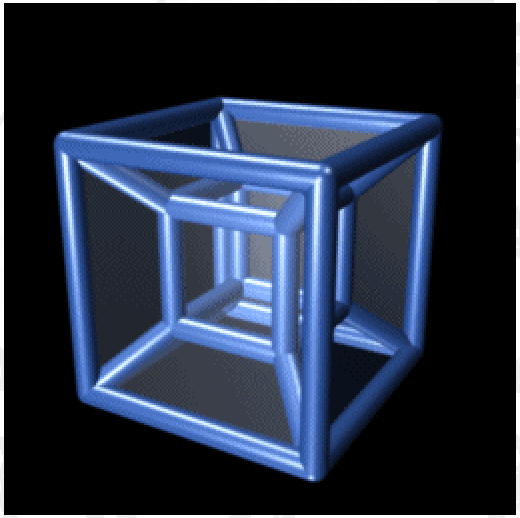
Loop returns to 3D space

Visualizing the hypercube

- The tesseract is the four-dimensional analog of the cube; the tesseract is to the cube as the cube is to the square





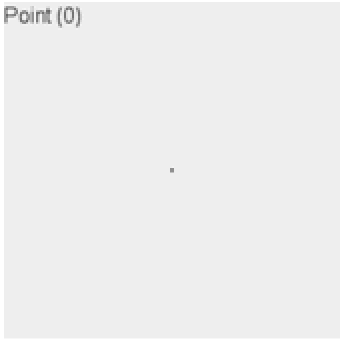


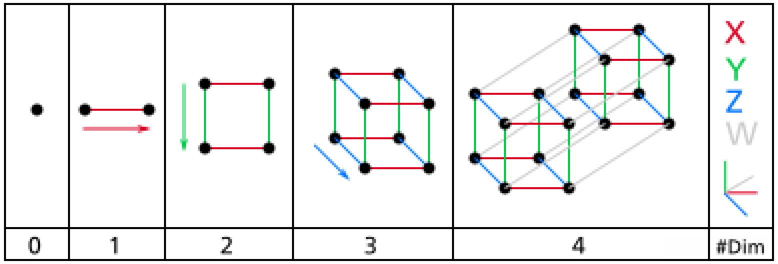
- Shine light on top of wireframe cube
- Picture of the shadow of a 3-cube
- Shadow of the 4-cube in motion

Visualizing the hypercube

- Drag the zero dimensional point and move it a distance of one unit in a (any) direction (outside 0-space), we form a unit line
- Drag the line and move it a distance of one unit in a direction perpendicular to the line (outside 1-space), this generates a unit square
- Drag the square and move it a distance of one unit in a direction perpendicular to the plane (outside 2-space), the result is a unit cube

Visualizing the hypercube

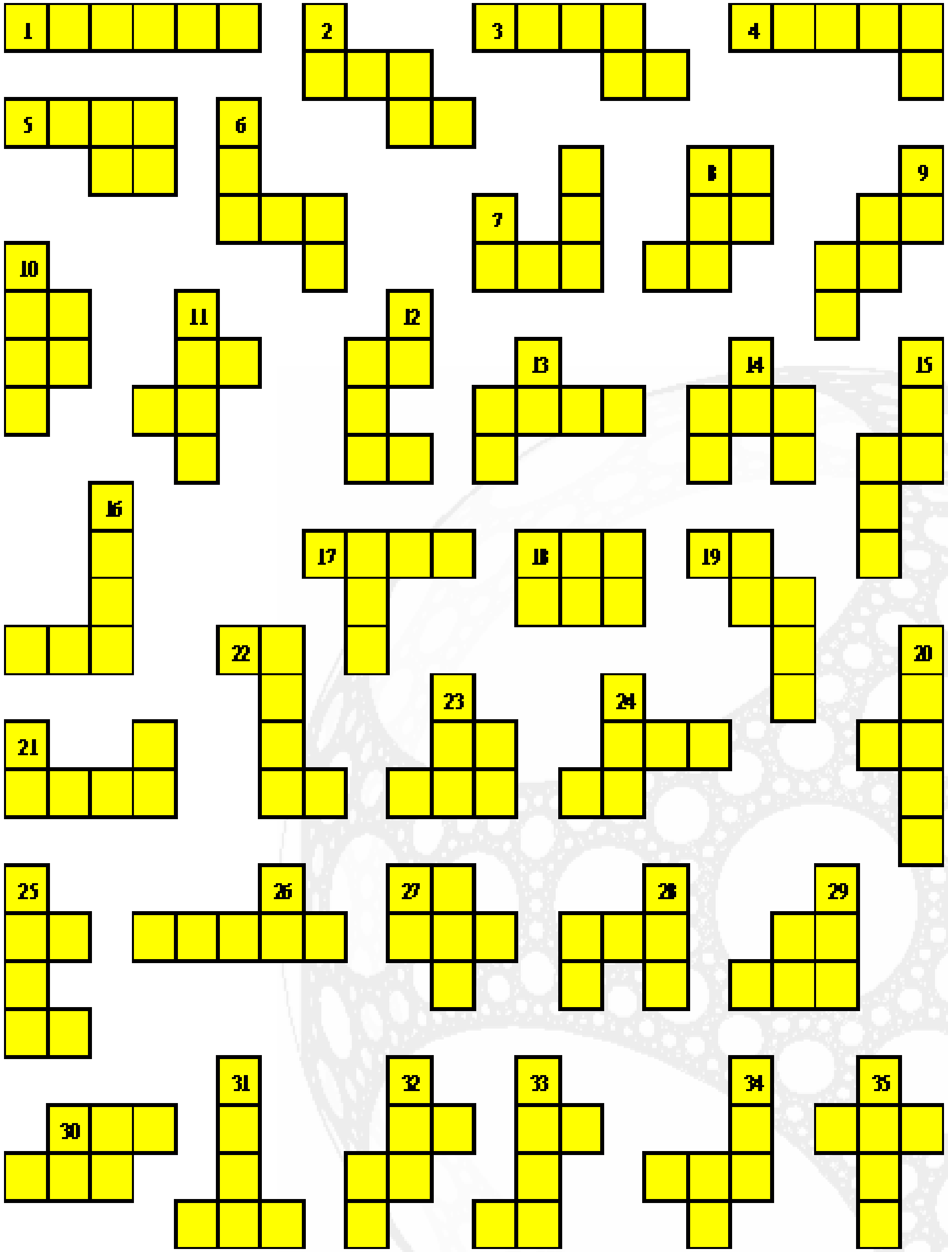




- Drag the cube and move it a distance of one unit in a direction perpendicular to all three axis of 3-space (outside 3-space) get a tesseract
- the object generated by such a shift is a 4-space unit hypercube with four mutually perpendicular edges meeting at every corner.
- Repeat this procedure with a 4-cube and you can get a 5-cube, and so on for any higher hypercubes.

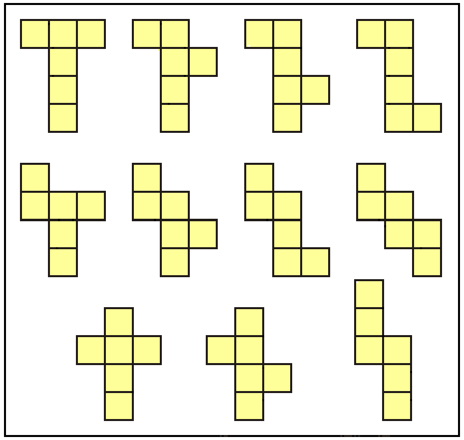
Hexominoes

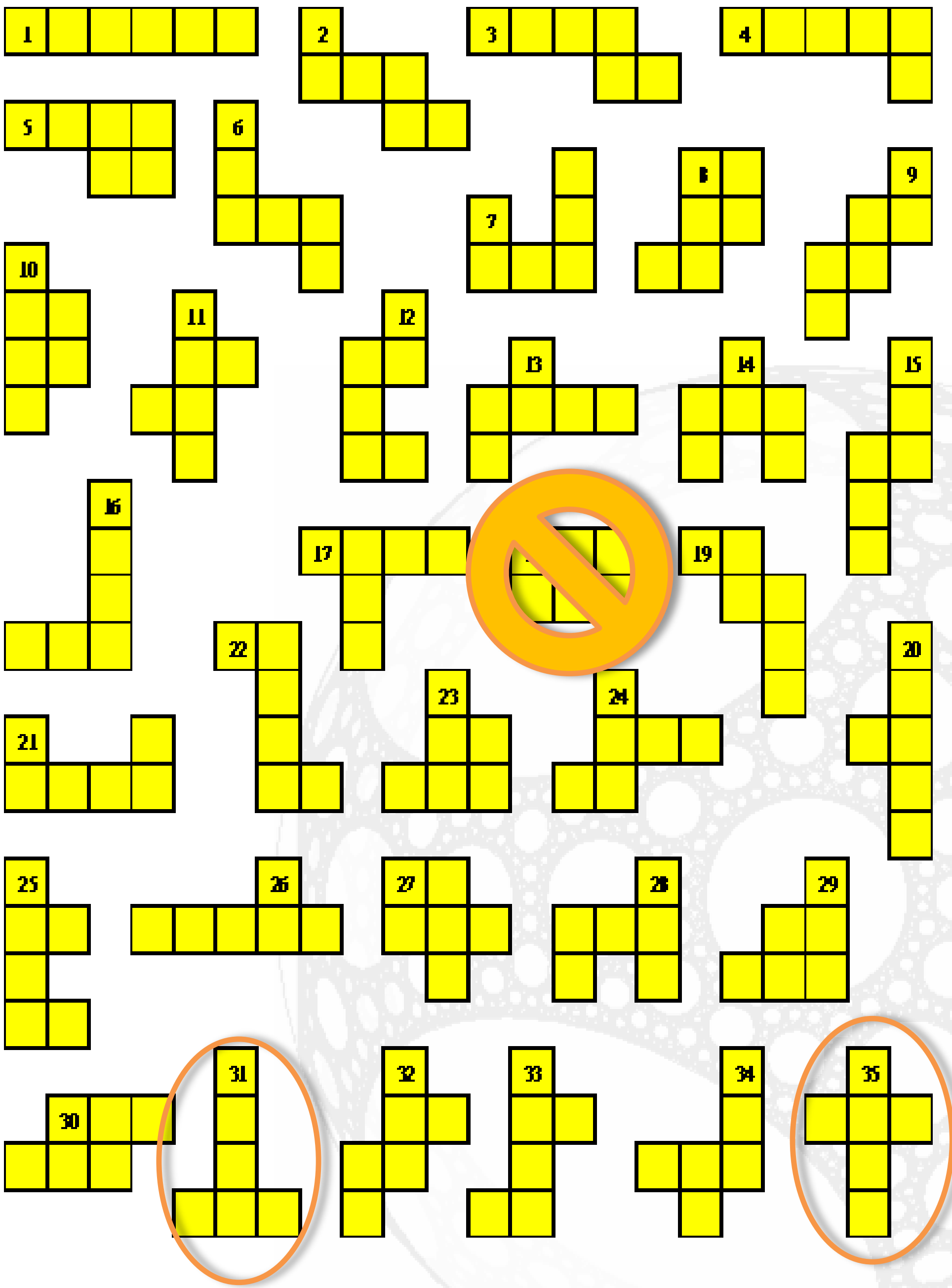
- Hexominos are figures with six squares joining together.
- Each square has at least one side touching another squares.
- There are 35 hexominos, omitting the mirror images or rotations of the figures.



Nets of the Cube

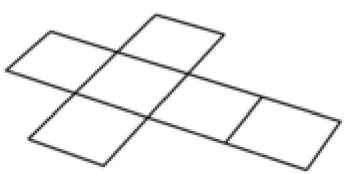
- Which of the above hexominos can fold to form a cube?
- There are 11 of them

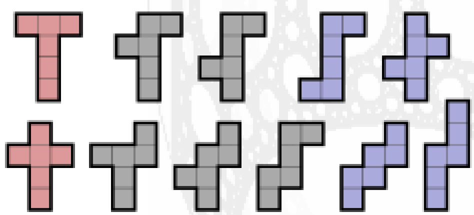




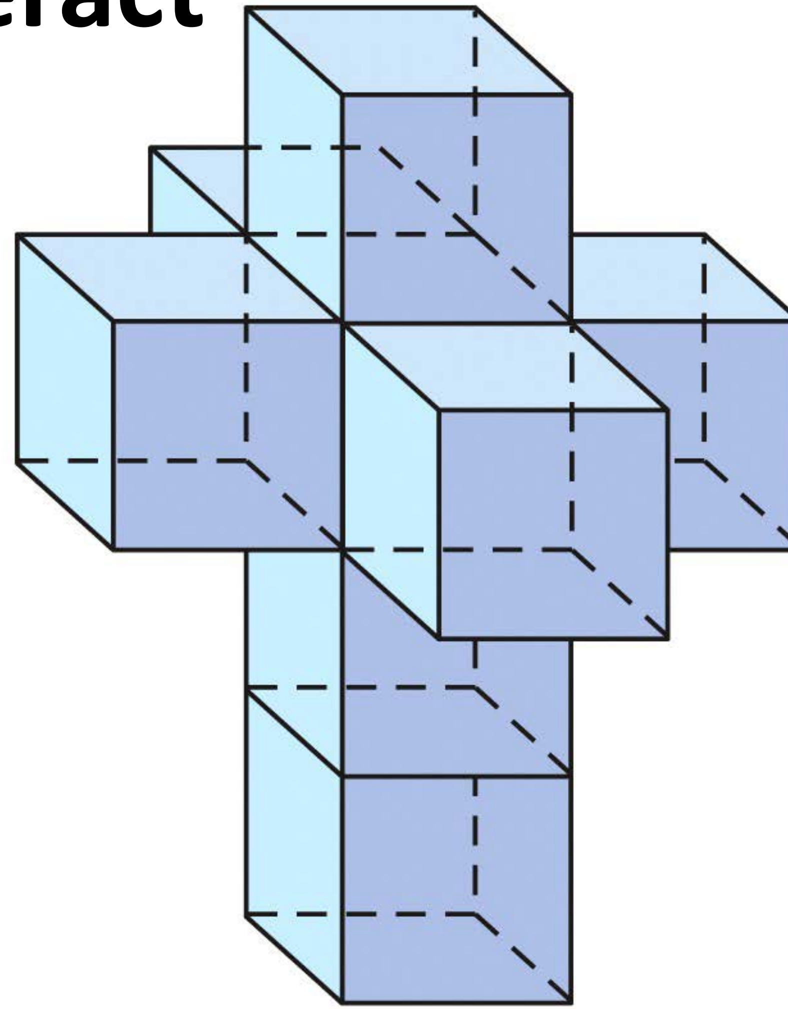
Nets of the Cube

- Which of the above hexominos can fold to form a cube?
- There are 11 of them
- https://www.geogebra.org/m/M5dZnUeH#material/RrknfdZz





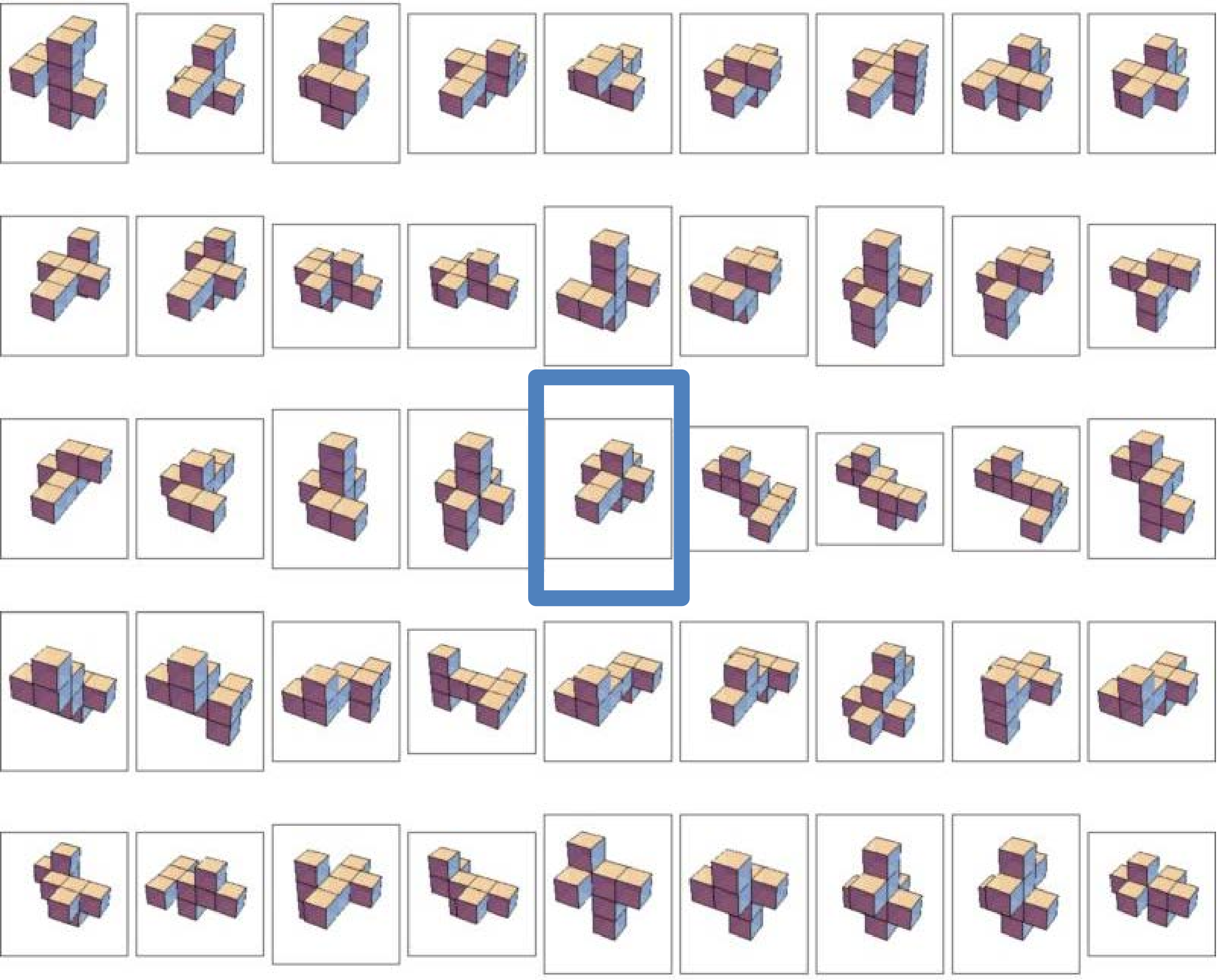
Nets of the tesseract

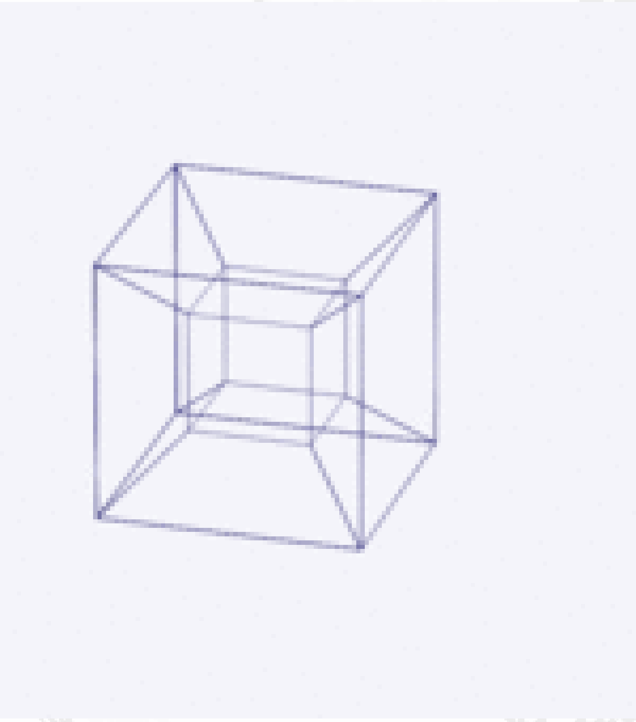


Unfolded 4D cube
in three dimensions

261 Nets of the tesseract

- https://mathoverflow.net/questions/198722/3d-models-of-the-unfoldings-of-the-hypercube?noredirect=1&lq=1

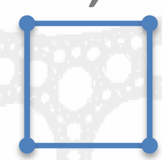




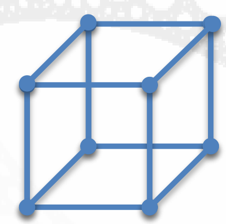
Elementary my dear Watson!

- Every n-cube of n > 0 is composed of elements, or n-cubes of a lower dimension, on the (n-1)-dimensional surface on the parent hypercube.
- 1-cube, the unit line, has 2 vertices

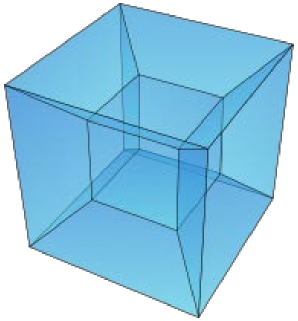
- 2-cube, the unit square, has 4 vertices, and 4 edges



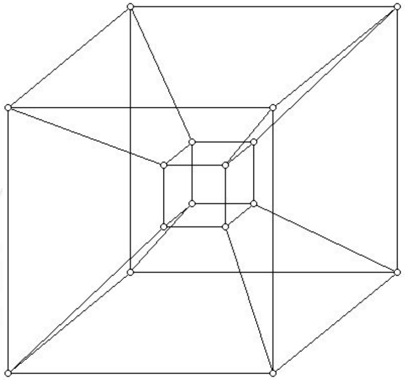
- 3-cube, the unit cube, has 8 vertices, 12 edges, and 6 square faces



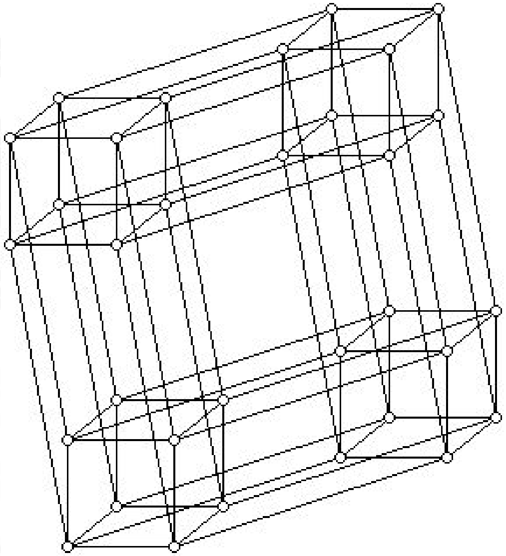
Tesseract: 4-cube



- a 4-cube is a four-dimensional hypercube with 16 vertices, 32 edges, 24 square faces, and 8 cubic cells

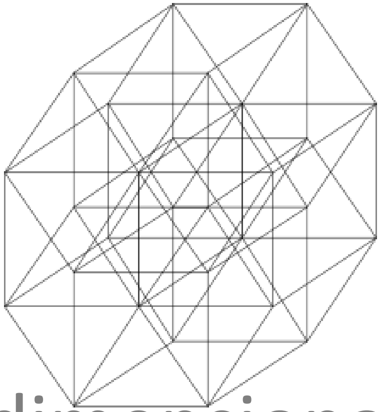


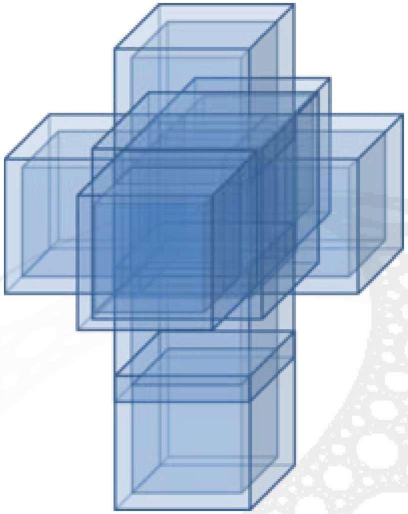
- Looks like a cube inside a cube with some connected vertices.
- Make a "line" of tesseracts, to make Penteract



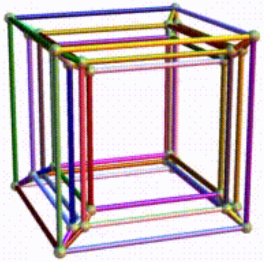
- One of the shadows of a 5-cube, where you can "see" a 4-cube inside a 4-cube with some connected vertices.

Penteract: 5-cube



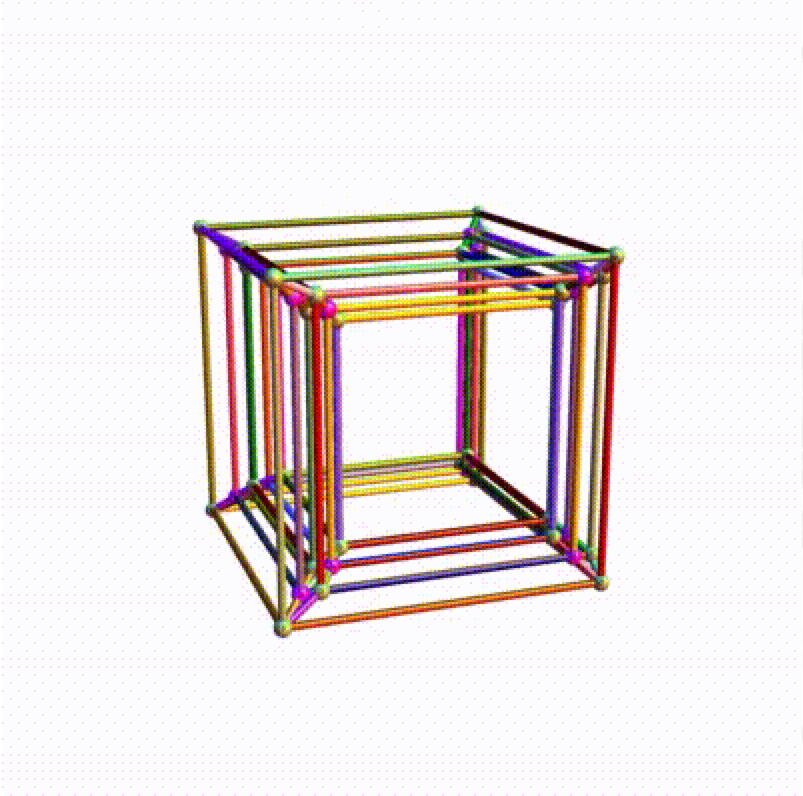


- 5-cube is a name for a five-dimensional hypercube with 32 vertices, 80 edges, 80 square faces, 40 cubic cells, and 10 tesseract 4-faces.
- 4D net of the 5-cube, perspective projected into 3D.
- Shadow of the 5-cube in motion



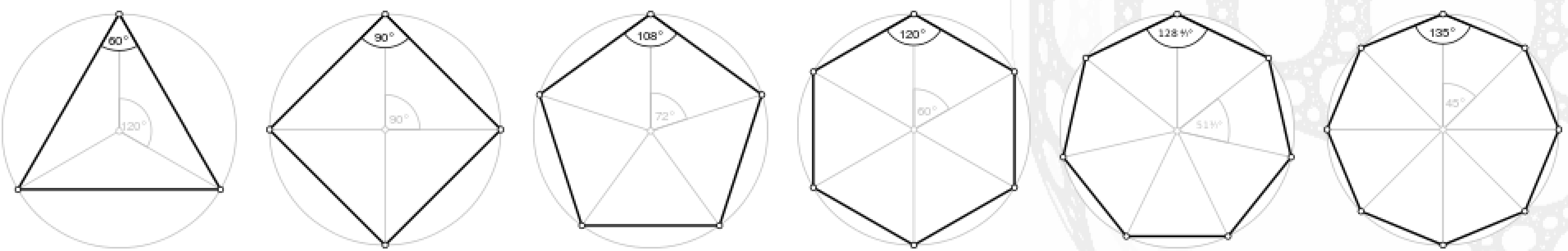
Hexeract: 6-cube

- a 6-cube is a six-dimensional hypercube with 64 vertices, 192 edges, 240 square faces, 160 cubic cells, 60 tesseract 4-faces, and 12 5-cube 5-faces.



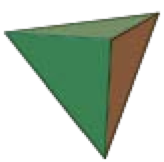
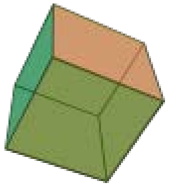
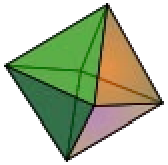
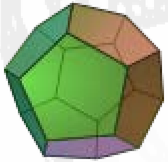
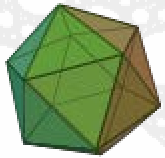
Road to Polytopes: Polygons

- In Euclidean geometry, a regular polygon is a polygon that is equiangular (all angles are equal in measure) and equilateral (all sides have the same length).
- For example, equilateral triangle, square, pentagon, hexagon, heptagon, octagon, etc.



Road to Polytopes: Platonic Solid

- In three-dimensional space, a Platonic solid is a regular, convex polyhedron. It is constructed by congruent regular polygonal faces with the same number of faces meeting at each vertex. Five solids meet those criteria:

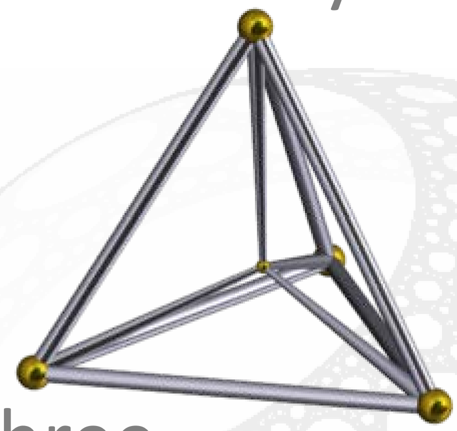
Tetrahedron	Cube	Octahedron	Dodecahedron	Icosahedron
Four faces	Six faces	Eight faces	Twelve faces	Twenty faces
				

Polytopes

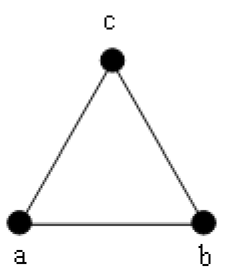
- A regular 4-polytope is a regular four-dimensional polytope. They are the four-dimensional analogs of the regular polyhedra in three dimensions and the regular polygons in two dimensions.
- Five of them may be thought of as close analogs of the Platonic solids. There is one additional figure, the 24-cell, which has no close three-dimensional equivalent.
- Each convex regular 4-polytope is bounded by a set of 3-dimensional cells which are all Platonic solids of the same type and size. These are fitted together along their respective faces in a regular fashion.

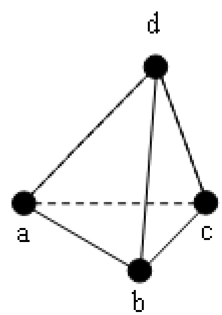
5-cell or 4-simplex

- 5-cell is a four-dimensional object bounded by 5 tetrahedral cells.

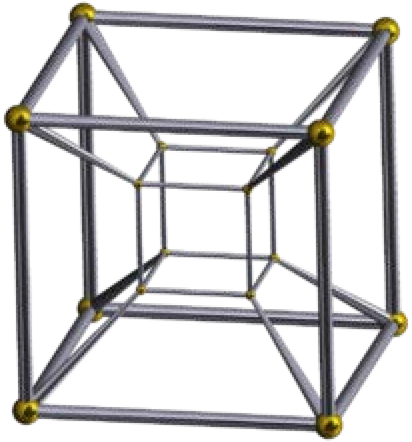


- is analogous to the tetrahedron in three dimensions and the triangle in two dimensions.



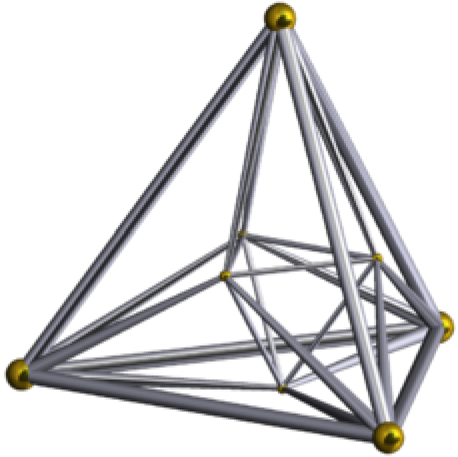


8-cell, Tesseract or 4-cube



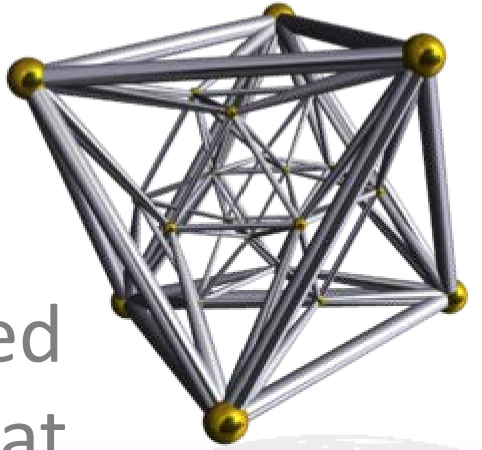
- The tesseract is the four-dimensional analog of the cube. The tesseract is to the cube as the cube is to the square.
- Three cubes and three squares intersect at each edge. There are four cubes, six squares, and four edges meeting at every vertex. All in all, it consists of 8 cubes, 24 squares, 32 edges, and 16 vertices.

16-Cell or hexadecachoron



- It is bounded by 16 cells, all of which are regular tetrahedra. It has 32 triangular faces, 24 edges, and 8 vertices.
- There are 8 tetrahedra, 12 triangles, and 6 edges meeting at every vertex.

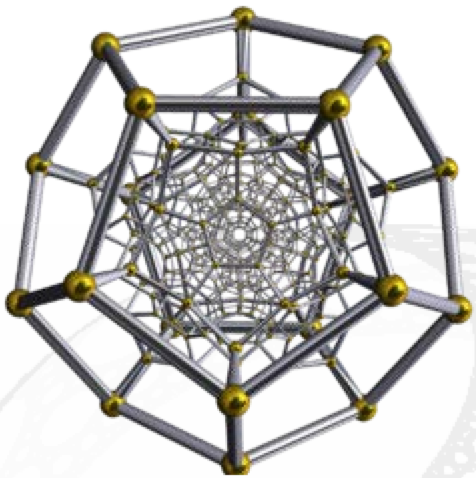
24-cell or icositetrachoron



- The boundary of the 24-cell is composed of 24 octahedral cells with six meeting at each vertex, and three at each edge. Together they have 96 triangular faces, 96 edges, and 24 vertices.
- the 24-cell is the unique convex self-dual regular Euclidean polytope which is neither a polygon nor a simplex
- It does not have a good analogue in 3 dimensions.

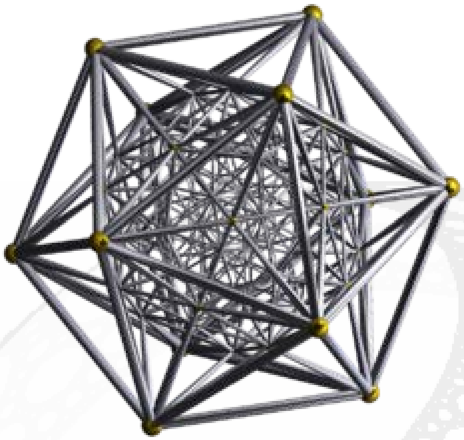
120-cell or hecatonicosachoron

- The 120-cell is regarded as the 4-dimensional analog of the dodecahedron, since it has three dodecahedra meeting at every edge, just as the dodecahedron has three pentagons meeting at every vertex.



- The boundary of the 120-cell is composed of 120 dodecahedral cells with 4 meeting at each vertex.

600-cell or hexacosichoron

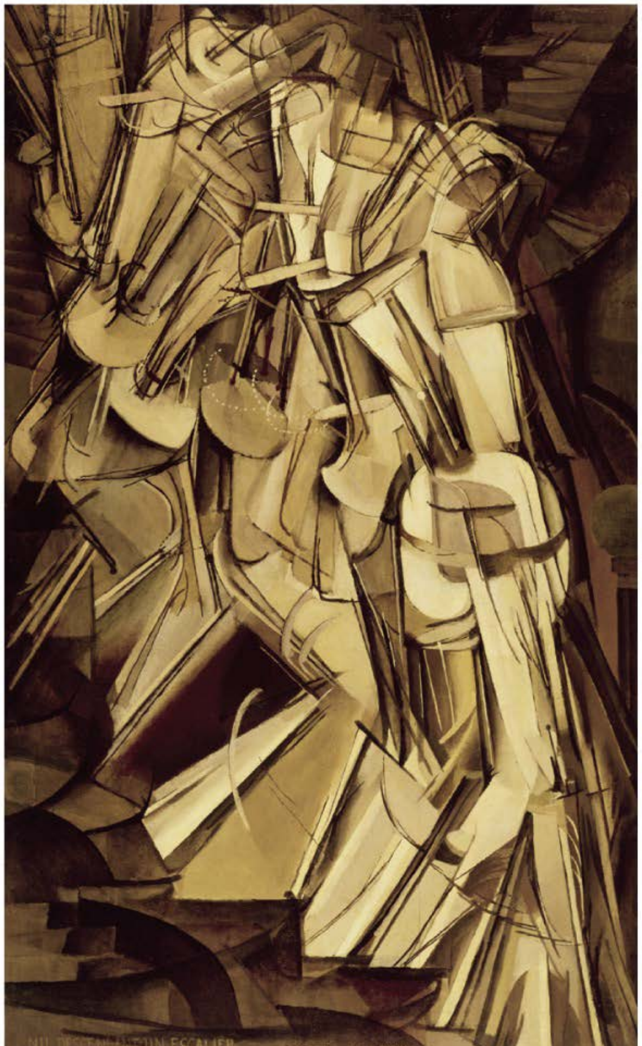


- The 600-cell is regarded as the 4-dimensional analog of the icosahedron, since it has five tetrahedra meeting at every edge, just as the icosahedron has five triangles meeting at every vertex.
- Its boundary is composed of 600 tetrahedral cells with 20 meeting at each vertex.

Hidden Dimensions: Exploring Hyperspace

- Watch the following forum from the World Science Festival, and dive deeper into these rich and beautiful subject
- https://www.youtube.com/watch?v=h9MS9i-CdfY&t=1788s

4th-dimension in art



The Knife Grinder by Kazimir Malevich (1912)

https://en.wikipedia.org/wiki/Fourth_dimension_in_art